SET THEORY AND ALGEBRA

SETS

1. SETS AND SUBSETS

Set: well-defined unorderd collection of distinct elements

Ex: $A = \{1,2,3,4\}$

$S = \{$set of all students in a class$\}$

Nullset: Set with No elements is called Null set, denoted as ϕ (or) $\{\}$

subset: If every ele of A is also an element of 'B' then A is subset of B.

$A = \{1,2,3\}$ $B = \{1,2\}$ Here $(B \subseteq A)$

Note: For every set 'A', 'A' and 'ϕ' are called Trivial subsets of 'A'.

Proper subset: Any subset of 'A' which is not a trivial subset is called proper subset of 'A'.

Note: If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

2. POWER SET

Denoted by $p(A)$

⇒ If 'A' is finite set then set of all finite subsets of 'A' is called power set of 'A'. It is denoted by $p(A)$.

Ex: $A = \{a,b\}$

subsets of $A = \phi, \{a\}, \{b\}, \{a,b\}$

$p(A) = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

⇒ If a set contains 'n' elements $(|A| = n)$ then $\underbrace{|p(A)|}_{\text{contains}} = 2^n$ elements.

3. COMPLIMENT AND DIFFERENCE

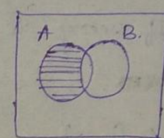
Universal set : set of all objects under discussion. denoted by 'U'

Compliment of a set : If 'A' is any set, then complement of 'A' denoted by $\overline{A}$ or A^c is called

$$A^c = \{x / x \notin A \text{ and } x \in U\}$$

Difference If 'A' and 'B' are two sets then

$$A - B = \{x / x \in A \text{ and } x \notin B\}$$



$A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$A - B = \{1,2\}$

4. UNION, INTERSECTION AND SYMMETRICAL DIFFERENCE

Set Intersection : $A \cap B = \{x / x \in A \text{ and } x \in B\}$

Set union : $A \cup B = \{x / x \in A \text{ OR } x \in B\}$

Note : If $A \cap B = \phi$, then 'A' and 'B' are disjoint sets.

Symmetric difference or Boolean sum : $A \Delta B$ / $A \oplus B$ $= \{x / x \in A \text{ or } x \in B \text{ but } x \notin A \cap B\}$.

$$\boxed{\begin{aligned} A \Delta B &= (A - B) \cup (B - A) \\ A \Delta B &= (A \cup B) - (A \cap B) \end{aligned}}$$

5. LAWS OF SETS

Commutative laws : (i) $A \cup B = B \cup A$

$A \cap B = B \cap A$

$A \oplus B = B \oplus A$

Associative laws :

1) $(A \cup B) \cup C = A \cup (B \cup C)$

2) $(A \cap B) \cap C = A \cap (B \cap C)$

3) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

Distributive la

1) $A \cup (B \cap C) = (A \cup$

2) $A \cap (B \cup C) = (A \cap$

Modular laws

1) $(A \cup B) \cap C = A \cup ($

2) $(A \cap B) \cup C = A \cap (B$

3) $A \cup \phi = A$ 5)

4) $A \cap \phi = \phi$ 6)

6. Example

Which of the

a) $A - (A - B) = B$

b) $A - (A - B) = ($

Use this diagra
3 sets

iii) $A - (A - B) = ($

$= \{2,3\} - \{2\} =$

(2)

Distributive laws:

1) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

2) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

Modular laws

1) $(A\cup B)\cap C = A\cup(B\cap C)$ iff $A \subseteq C$

2) $(A\cap B)\cup C = A\cap(B\cup C)$ iff $C \subseteq A$

3) $A\cup\phi = A$

4) $A\cap\phi = \phi$

5) $A\cup U = U$, $A\cap U = A$

6) $A\cup A^c = U$ $\quad A\cap A^c = \phi$

Demorgan law

1) $(A\cup B)^c = A^c \cap B^c$

2) $(A\cap B)^c = A^c \cap B^c$

3) $A-(B\cup C) = (A-B)\cup(A-C)$

4) $A-(B\cap C) = (A-B)\cap(A-C)$

Idempotent law

1) $A\cup A = A$

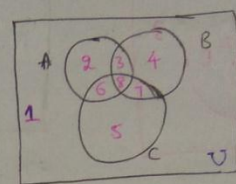
2) $B\cap B = B$

Absorption law

1) $A\cup(A\cap B) = A$

2) $A\cap(A\cup B) = A$

(3)



$A\cup(B\cap C) = \{2,3,6,8\}\cup\{8,7\}$

$= \{2,3,6,7,8\}$

$(A\cup B)\cap(A\cup C) = \{2,3,4,6,7,8\}\cap$

$\{2,3,6,8,7,5\}$

$= \{2,3,6,8,7\}$

6. EXAMPLE 1

Which of the following is not TRUE?

a) $A-(A-B) = B$

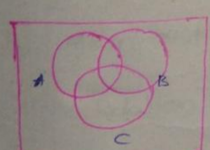
b) $A-(A-B) = (A\cap B)$

(c) $(A\cap B)\cup(A\cap B^c) = A$

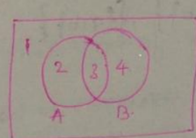
(d) $B\cap(A\cup B) = A$

No. of Regions $= 2^n$

$n =$ no. of sets.



Use this diagram for 3 sets



for two sets.

i) $A\ \{2,3\} - \{2\} = \{3\}$

$\times\ B = \{3,4\}$

ii) $(A\cap B)\cup(A\cap B^c) = A$

$= \{3\}\cup\{\{2,3\}\cap\{1,2\}\}$

$= \{3\}\cup\{2\} = \{2,3\}$

$= A$.

iii) $A-(A-B) = (A\cap B)$

$= \{2,3\} - \{2\} = \{3\} = A\cap B$.

(iv) $B\cap(A\cup B) = A$

$= \{3,4\}\cap\{2,3,4\}$

$= \{3,4\} \neq A$.

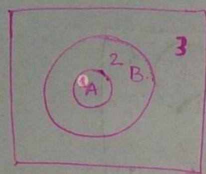
$=$ False

7. EXAMPLE - 2

Which of the following is not True ?

a) If $A \subset B$, then $B^c \subseteq A^c$ ✓ (c) $A \cap p(A) = \phi$ ✓

b) $A \cap p(A) = A$ ✗ (d) $p(A) \cap p(p(A)) = \{\phi\}$ ✓



1> $B^c = \{3\}$, $A^c = \{2, 3\}$ $\longrightarrow$ $B^c \subset A^c$ (TRUE)

2> $A \cap p(A) = \phi$

These are elements ← $A = \{a, b\}$

These are sets ← $p(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

$A \cap p(A) = \{\cancel{\phi}\}$

$\therefore$ so $\boxed{A \cap p(A) = \phi}$

3> $p(A) \cap p(p(A)) = \{\phi\}$

$p(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

$p(p(A)) = \{\phi, \{\{a\}\}, \{\{b\}\}, \{\{a, b\}\}, \ldots\}$

8. EXAMPLE - 3

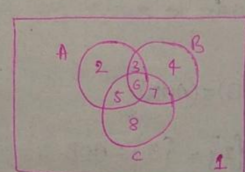
Which of the following is NOT TRUE ?

a) $(A-B)-C = (A-C)-B$ ✓ = TRUE

b) $(A-B)-C = (A-C)-(B-C)$ ✓ = TRUE

c) $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$ ✗ = FALSE

d) $A - (B \cup C) = (A-B) \cap (A-C)$ ✓ = TRUE



1) $(A-B) = \{2, 3, 5, 6\} - \{3, 6, 7, 4\} = \{2, 5\} - \{5, 6, 7, 8\}$
$= \{2\}$

(a) $(A-C)-B = \{2, 3\} - \{3, 6, 7, 4\} = \{2\}$

(8) $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$

$\{2, 3, 5, 6\} \oplus \{3, 4, 6, 7, 5, 8\}$

present here but not here ($\downarrow$) and present here but not here

$= \{2\} \{2, 4, 7, 8\}$

NOW, $(A \oplus B) = (2, 5, 4, 7) \cup (2, 3, 7, 8) = (2, 3, 4, 7, 8) \neq \{2, 4, 7, 8\}$

2. RELATIONS

1. INTRODUCTION TO RELATIONS

Cartesian Product: $A = \{1,2,3\}$ $B = \{a,b\}$

$A \times B = \{(1,a)(1,b)(2,a)(2,b)(3,a)(3,b)\}$
↳ Each ele is called Orderd pair.

⇒ cartesian product = cross product

⇒ Here $|A \times B| = 3 \times 2 = 6$.

⇒ In general if $|A| = m$, $|B| = n$ then $(A \times B)$ contains $(m \times n)$ orderd pairs

⇒ we take a Relation from cartesian product.

Relation:

The Relation is a subset of cartesian product

For above example $R = \{(1,a)(1,b)\}$

$1Ra \Rightarrow (1,a) \in R$

$R' = \{(1,a)(2,a)(3,a)\}$

$R' = \{1R'a, 2R'a, 3R'a\}$

⇒ If $\boxed{|A| = m \text{ and } |B| = n \text{ then no. of Relations that can be formed} = 2^{m \times n}}$

(Since Relation is subset of cartesian product, and cross product contains $(m \times n)$ elements. Now, the no. of subsets that are possible over the cartesian product set is $2^{m \times n}$, so the no. of relations will be $2^{m \times n}$.)

Ex-1 Let $R = \{(x,y) \in Z \times Z : x < y\} = \{(1,2)(2,3)(4,5) \ldots\}$

Ex-2: Let $R = \{(x,y) \in Z \times Z : (x+y) \text{ is even}\} = \{(1,3)(1,5)(2,4)(6,4) \ldots\}$

Ex-3: Let $A = \{1,2,3,4\}$ $B = \{1,2\}$

$R : \{(x,y) \in A \times B : x+y = 3\} = \{(1,2)(2,1)\}$

2. REFLEXIVE RELATION

Reflexive Relation: A Relation 'R' on set 'A' is said to be Reflexive if $(xRx)\ \forall x \in A$

$A = \{1,2,3\}$

$A \times A = \{(1,1)(1,2)(1,3)$
$(2,1)(2,2)(2,3)$
$(3,1)(3,2)(3,3)\}$

$R = \{(1,1)(2,2)(3,3)\}$ Here 'R' is Reflexive Relation

$R_1 = \{(1,1)(2,2)\}$ is not Reflexive because it doesnot contain $(3,3)$ ($\forall x \in A$ xRx should be in R)

$R_2 = \{(1,1)(2,2)(3,3)(1,2)\}$ = Reflexive Relation

$\therefore$ Let $A = \{(1,2 --- n)\} \rightarrow n$ elements

$R_n = \{(1,1)(2,2) --- (n,n)\} \rightarrow$ Then the smallest Relation which is reflexive contains 'n' elements.

Note: If 'R' is Reflexive then any superset of 'R' is Reflexive

The largest Reflexive Relation on 'A' is "A×A"

The smallest Reflexive Relation on 'A' contains 'n' elements

If
$\lvert A\rvert = n$	then	largest Reflexive Relation $= n \times n = n^2$ elements.
$\lvert A\rvert = n$	then	smallest Reflexive Relation $= n$ elements.

3. EXAMPLE 1 ON REFLEXIVE RELATIONS

If $A = \{1,2,3 --- n\}$ then the no. of reflexive relations possible on 'A'?

$A = \{1,2\}$

$R_1 = \{(1,1)(2,2)\}$ ✓

$R_2 = \{(1,1)(2,2)(1,2)\}$ ✓

$R_3 = \{(1,1)(2,2)(2,1)\}$ ✓

$R_4 = \{(1,1)(2,2)(2,1)(1,2)\} = A \times A$ ✓

$A \times A = \{(1,1)(1,2)(2,1)(2,2)\}$ → we have 2 choices
we may or may not include it.

we have one choice

$A = \{1,2,3\}$

$R_1 = \{(1,1)(2,$

$\therefore$ If $\lvert A\rvert = n$

$\lvert A\rvert = n$

The NO. of Rel

Non-

4. EXAMPLE 2

Now whenever
an element for
element 2 times.

Ex:

1> The relation

2> The relation
Real number

3> The Relation
collection of

4> The Relation

5> $R: \{(x,y) \in$

$A = \{1,2,3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$ → 1 way

$\left.\begin{matrix} \underline{2} & \underline{2} & \underline{2} \\ \underline{2} & \underline{2} & \underline{2} \end{matrix}\right\} = 2^6$ combinations

∴ The no. of Reflexive relations $= 64$.

∴ If $|A| = n$ then the no. of Reflexive Relations on $A = 2^{n^2-n}$

$$\boxed{|A| = n \Rightarrow \text{NO. of Reflexive Relations on } A = 2^{n(n-1)}}$$

The No. of Relations that are not Reflexive = Total Relations − Reflexive Relations

$$= 2^{n \times n} - 2^{n^2-n}$$

$$\boxed{\text{Non-Reflexive Relations from A to A} = 2^{n^2} - 2^{n^2-n}}$$

4. EXAMPLE 2 ON REFLEXIVE RELATIONS

Now whenever they ask if a Relation is Reflexive then try to take an element from the relation and apply the condition between the same element 2 times.

Ex:

1) The relation $\leq$ is reflexive on any set of Real numbers $(x \leq x)$

2) The relation 'is a divisor of' is reflexive on a set of non-Zero Real numbers (x/x) x is divisible by 'x' = TRUE ⇒ Relation is Reflexive

3) The Relation is a subset of denoted by '$\subseteq$' is reflexive on any collection collection of sets $(A \subseteq A) \Rightarrow$ TRUE ⇒ Reflexive

4) The Relation 'is parallel to' is reflexive on a set of all lines $(L \parallel L)$ = TRUE

5) $R: \{(x,y) \in Z \times Z : x-y \text{ is even integer}\}$ $(x-x) = 0$ = even = TRUE ⇒ Reflexive

5. EXAMPLE 3 ON REFLEXIVE RELATIONS

Which of the following is False?

a) If R_1 is Reflexive then every superset of R_1 is Reflexive = TRUE

b) If R_1 is Reflexive, then subset of R_1 is reflexive = FALSE

c) If R_1, R_2 are Reflexive then $R_1 \cap R_2$ is Reflexive = TRUE

d) If $R_1 R_2$ are Reflexive then $R_1 \cup R_2$ is Reflexive = TRUE

A.

$R = \{(1), 2, 3\}$

$R_1 = \{(1,1)(2,2)(3,3)\}$.

$R_1 = \{(1,1)(2,2)(3,3)(1,2)(1,3)$
$(2,3)(3,1)\}$ are reflexive

B. $R = \{1,2,3\}$.

$R_1 = \{(1,1)(2,2)(3,3)\}$

$R_2 = \{(1,1)\} \rightarrow$ Not Reflexive

C.

$R_1 = \{(1,1)(2,2)(3,3)\}$

$R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,3)\}$

$R_1 \cap R_2 = \{(1,1)(2,2)(3,3)\}$
= Reflexive.

D. $R_1 = \{(1,1)(2,2)(3,3)\}$.

$R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,3)\}$

$R_1 \cup R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,3)\}$
= Reflexive

6. IRREFLEXIVE RELATIONS

Irreflexive Relations:

The Relation 'R' on a set 'A' is called irreflexive if 'x' is not related to 'x' i.e $x \not R x \ \forall x \in A$ i.e the orderd pair $(x,x) \notin R \ \forall x \in A$

Ex:

If a Relation is Not reflexive then we think that it is irreflexive but this is wrong assumption.

Ex: $A = \{1,2,3\}$

$R_1 = \{(1,1)(2,2)(3,3)\}$ Reflexive, NOT Irreflexive

$R_2 = \{\ \}$ NOT Reflexive, Irreflexive

$R_3 = \{(1,1)\}$ NOT Reflexive, Not Irreflexive

$R_4 = \{(1,2)(2,1)\}$ NOT Reflexive, Irreflexive

There does not exist a relation which is both Reflexive and Irreflexive

⇒ There will be some relations which are neither reflexive nor Irreflexive

9 (85)

⇒ The min. set that is Irreflexive is { } and the min cardinality of a Irreflexive Relation is 'Zero'.

Now, $R_5 = A \times A$

$= \{(1,1)\ (1,2)\ (1,3)$
$(2,1)\ (2,2)\ (2,3)$
$(3,1)\ (3,2)\ (3,3)\}$

Now, Remove the diagnol elements because they make the Relation Reflexive.

Now $\{(A \times A) - (\text{diagnol elements})\} = \{(1,2)(1,3)$
$(2,1)(2,3)$
$(3,1)(3,2)\}$ This is the largest Irreflexive Relations

$\boxed{\text{If } |A| = n \text{ the then the cardinality of largest Irreflexive Relation will be } (n^2 - n)}$

$\{\text{Total no. of elements in } A \times A = (n \times n) = n^2\} - \{\text{Diagnol elements}\}$

7. EXAMPLE 1 ON IRREFLEXIVE RELATIONS

Now, I want to find no. of Irreflexive Relations on 'A' given $|A| = n$

⇒ then $\boxed{|A \times A| \text{ contains } n^2 \text{ elements}}$

let $A = \{1,2,3\}$

$A \times A = \{(1,1)\ (1,2)\ (1,3)$
$(2,1)\ (2,2)\ (2,3)$
$(3,1)\ (3,2)\ (3,3)\}$

→ 0 ways (they should not be included)

2 2 2 2 2 2

No. of Irreflexive Relations $= 2^6 = 64$

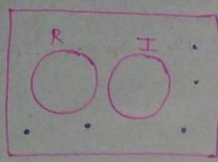
∴ In general $A \times A = \{\underbrace{(1,1)\ (2,2)(3,3) --- (n,n)}_{'n' \text{ ele}}, \overbrace{(1,2)\ (1,3) ---}^{n^2 \text{ elements}}\}$

2 0

$n^2 - n$ ele

∴ $\boxed{\text{The total no. of Irreflexive relations that can be formed over a set of 'n' elements is } = 2^{n^2 - n}}$

Now, it is clear that



R = Reflexive Relations.

I = Irreflexive Relations

Now, No. of Relations which are either reflexive or irreflexive is $= 2^{n^2-n} + 2^{n^2-n}$

$= 2\left(2^{n^2-n}\right)$

$= 2^{n^2-n+1}$ //

$\therefore$ $\boxed{\text{No. of Relations that are either Reflexive or Irreflexive } 2^{n^2-n+1}}$

Now, No. of Relations that are neither Reflexive nor Irreflexive = (Total) − (No. of Reflexive + Irreflexive)

$$= 2^{n^2} - \left(2^{n^2-n} + 2^{n^2-n}\right)$$

$\therefore$ $\boxed{\text{No. of Relations that are Neither reflexive nor Irreflexive } = 2^{n^2} - \left(2^{n^2-n+1}\right)}$

8. EXAMPLE 2 ON IRREFLEXIVE RELATIONS

→ The Relation '<' on set of all Real numbers is Irreflexive = TRUE

→ The Relation '⊂' (→ proper subset) on set of all sets is "irreflexive" = A ⊂ A = TRUE

→ The Relation ⊥ on set of all lines is "Irreflexive" = $x \perp x$ (A line x, x will be parllel not ⊥r ∴ Two ele (x, x) will never belong to this relation.

⇒ Fore the above examples choose a value and check the condition if it is valid then the Relation is Not IRReflexive if it fails then it is Irreflexive.

9. EXAMPLE 3 ON IRREFLEXIVE RELATION

→ which of the following is false?

→ Every subset of Irreflexive relation is irreflexive

→ Every superset of Irreflexive relation is Irreflexive

→ If R_1 is Irreflexive, R_2 is irreflexive then $R_1 \cap R_2$ is irreflexive

→ If R_1, R_2 are irreflexive then $R_1 \cup R_2$ is Irreflexive.

Ans

A) $A = \{1,2,3\}$

$R = \{(1,2)(1,3)$
$(2,3)(2,1)$
$(3,1)(3,2)\}$

R_1 = Subset of R
$= \{(1,2)(1,3)\}$ = IRREFLEXIVE
TRUE

(B)

$A = \{1,2,3\}$

$R = \{(1,2)(1,3)(2,3)$
$(2,1)(3,1)(3,2)$
$(1,1)(2,2)\}$

$\downarrow$ Super Set of IR-Relation

= NOT IRREFLEXIVE

(C)

$A = \{(1),2,3\}$

$R_1 = \{(1,2)(1,3)\}$ $R_2 = \{(2,1)(2,3)\}$

$\Rightarrow R_1 \cap R_2 = \{\ \} = \phi$
= IRREFLEXIVE

(D) TRUE (use same example)

10. EXAMPLE ON CLOSURE PROPERITIES

State TRUE/ FALSE

a) The set of all reflexive relations set closed under the operation set union.
b) " " " " " " " " " " Intersection
c) " " " " " " " " " " difference
d) The set of all irreflexive Relations are closed under the operation set union
e) " " " " " " " " " " Intersection
f) " " " " " " " " " " difference

Set of all Reflexive Relations

11. SYMETRIC RELATIONS

A Relation 'R' on a set 'A' is said to be symetric if (xRy) then (yRx) $\forall x,y \in A$ i,e if the orderd pair $(x,y) \in R$ then $(y,x) \in R$ $\forall x,y \in A$.

$A = \{1,2,3\}$

$R_1 = \{(1,2)(2,1)\}$ → Symetric ✓
Reflexive ×
Irreflexive ✓

$R_2 = \{(1,1)\}$ – Symetric
Not Reflexive
Not Irreflexive

$R_3 = \{(2,3)\}$ → Not symetric
Not Reflexive
Irreflexive

$R_4 = \{(3,1)(1,1)\}$ – Not Symetric
– Not Reflexive
– Not Irreflexive

$R_5 = \{\}$ - symetric

The cardinality of the smallest symetric Relation is Zero

$R_6 = A \times A$ is symetric $\therefore$ The largest cardinality is "n^2" elements and

The largest symetric relation that is defined on a set 'A' is "$A \times A$"

12. NUMBER OF SYMETRIC RELATIONS

$|A| = n$ then how many Relations are possible that are symetric on $A \times A$

$|A| = n$ then $|A \times A| = n^2$

$A = \{1,2,3\}$

$$A \times A = \{\underbrace{(1,1)}_{2}\underbrace{(2,2)}_{2}\underbrace{(3,3)}_{2}\underbrace{\boxed{(1,2)(2,1)}}_{2}\underbrace{\boxed{(1,3)(3,1)}}_{2}\underbrace{\boxed{(2,3)(3,2)}}_{2}\} = 2^6$$

$|A| = n = \{1,2,3 --- n\}$.

$$(A \times A) = \overbrace{\{\underbrace{(1,1)(2,2)(3,3) --- (n,n)}_{n \text{ elements}}, \underbrace{\boxed{(1,2)\ (2,1)}\ \boxed{(2,3)(3,2)}}_{n^2 - n \text{ elements}}\}}^{n^2 \text{ elements}}$$

$= n^2$ ele

$2 \times 2 \times 2 \times \quad \times 2 = 2^n$

$2 \times 2 \ldots = 2^{(n^2-n/2)}$ → No. of pairs that will be formed.

$= 2^{(n^2-n)/2}$.

$$\boxed{\therefore \text{ No. of symetric Relations} = 2^n \times 2^{(n^2-n)/2} = 2^{n + \left(\frac{n^2-n}{2}\right)}}$$

13. EXAMPLE 1 ON SYMETRIC RELATIONS

$|A| = n$, now I want to find the relation between the no. of Symetric Relations and Reflexive Relations.

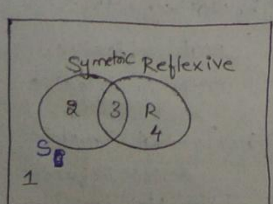
Let $A = \{1,2,3\}$

$R_1 = \{(1,2)(2,1)\}$

$R_2 = \{(1,1)(2,2)(3,3)\}$

$R_3 = \{(1,1)(2,2)(3,3)(1,2)\}$

$R_4 = \{(1,2)\}$



$\underbrace{2^{A \times A}}$ → subset of powerset (Total no. of relations).

(12)

W.K.T the no. of symetric Relations possible are $2^n \times 2^{(n^2-n)/2} = 2^{\frac{n(n+1)}{2}}$

Now, set of all Relations that are both Symetric and Reflexive = Region 3 $= 2^{n(n-1)/2}$

$$AXA = \{\underbrace{\overbrace{(1,1)(2,2)(3,3)---(n,n)\boxed{(1,2)(2,1)}----}^{n^2 \text{ elements}}}\}$$

1 way because the Relation should be reflexive (n elements).

n^2-n ele

$2^{(n^2-n)/2}$.

(13)

$$\therefore \boxed{|S \cap R| = 2^{\frac{n(n-1)}{2}}} \leftarrow \text{Symetric and Reflexive}$$

Now, $|S-R| = |n(S)| - n(S\cap R)|$

$= 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$

$\therefore$ $|S-R|$ = Symetric but not Reflexive $= 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$

Now, $|R-S| = n|R| - n|S \cap R|$

$$\boxed{|R-S| = 2^{n(n-1)} - 2^{\frac{n(n-1)}{2}}}$$

Now the Relations that are not Symetric and Reflexive $= n|\overline{S \cup R}|$

$= 2^{n^2} - (n(S) + n(R) - n(S \cap R))$

$= 2 - \left(2^{\frac{n(n+1)}{2}} + 2^{n(n-1)} - \left(2^{\frac{n(n-1)}{2}}\right)\right)$

$\therefore$ The relations that are not both Reflexive and Symetric is $= 2^{n^2} - \left(2^{\frac{n(n+1)}{2}} + 2^{n^2-n} - \left(2^{\frac{n(n-1)}{2}}\right)\right)$

15. EXAMPLE 3 ON SYMETRIC RELATIONS

State which of the following are TRUE and False?

a) Every subset of symentric Relation is symetric (F)

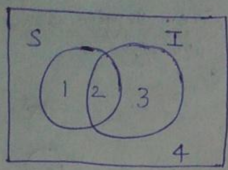
b) " superset " " " " (FALSE)

c) If R_1 and R_2 are symetric then $R_1 \cap R_2$ is symetric (TRUE)

d) " " " " " " " $R_1 \cup R_2$ " " (TRUE)

e) " " " " " " " $R_1 - R_2$ " " (TRUE).

16. RELATION BETWEEN SYMETRIC AND IRREFLEXIVE RELATION



$R_1 = \{(1,1)\} \rightarrow$ Symetric ✓ Not IReflexive

$R_2 = \{(1,2)(2,1)\} \rightarrow$ Symetric ✓ Irreflexive ✓

$R_3 = \{(1,2)\} \rightarrow$ Symetric ✗ Irreflexive ✓

$R_4 = \{(1,2)(1,1)\} \rightarrow$ Symetric ✗ Irreflexive ✗

WKT $n(S) = 2^{\frac{n(n+1)}{2}}$

$n(I) = 2^{n^2-n}$

Now, $n|S \cap I|$ = Both Symetric and Irreflexive

$$A \times A = \{\underbrace{(1,1)(2,2) \;---\; (n,n)}_{\times}\ \boxed{(1,2)(2,1)}\ ---\}.$$

$$2 \quad 2 \quad 2 \;--\; \tfrac{(n^2-n)}{2} \text{ times}$$

$\therefore \boxed{n|S \cap I| = 2^{(n^2-n)/2}}$

$n(S \cup I) = n(S) + n(I) - n(S \cap I) \rightarrow$ Either Symetric or Reflexive

$\therefore$ The no. of Relations that are Either Symetric or Irreflexive $= \left(2^{\frac{n(n+1)}{2}} + 2^{n^2-n}\right) - \left(2^{(n^2-n)/2}\right)$

$n(S-I) = n(S) - n(S \cap I)$

$\Rightarrow$ The no. of Relations that are symetric but not Irreflexive $= 2^{\frac{n(n+1)}{2}} - 2^{(n^2-n)/2}$

$n(I-S) = n(I) - n(S \cap I)$

$\Rightarrow$ The no of Relations that are but not symetric is $= 2^{n^2-n} - 2^{(n^2-n)/2}$

$n(\overline{I \cup S})$ = Relations that are neither Irreflexive nor Symetric $= n(U) - n(I \cup S) = 2^{n^2} - n(I \cup S)$

17. ANTIS

(14) A Relation

$\forall x, y \in A$.

$A = \{1, 2,$

$\therefore$ The min ca

Now, $A \times A = \{(1,1)$

$(1,2)$

$(2,3$

$(3,1$

Now, if $A \times A =$

$\therefore$ The max ca

17. ANTISYMETRIC RELATION

(14) A Relation 'R' is said to be Antisymetric if (xRy and yRx) then $x=y$ $\forall x,y \in A$.

$A = \{1,2,3\}$

$R_1 = \{(1,2)\ (2,1)\}$ → NOT Antisymetric because we have if $(x,y) \in R$ then $(y,x) \in R$.

$R_2 = \{(1,1)\}$ → Antisymetric (This is the only exception (the pairs of type (x,x) are allowed).

symetric ✓ (we cant say symetric & Antisymetric are compliment to eachother).

$R_3 = \{(1,2)\ (1,1)\}$ ⇒ Not symetric but it is Antisymetric

$R_4 = \{(1,2)(2,1)\ (2,3)\}$ ⇒ Not Antisymetric. $(x,y) \in R$ & $(y,x) \in R$.
Not Symetric $(3,2)$ is not present.

$R_5 = \{(1,1)\ (2,2)(3,3)\}$ → Both Symetric and Antisymetric (Exceptional case).

$R_6 = \{\ \}$ – Antisymetric ✓

∴ $\boxed{\text{The min cardinality of Antisymetric Relation} = 0}$

$R_7 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\ (3,1)\}$ = largest Antisymetric Relation

Now, $A \times A = \{(1,1)\ (2,2)\ (3,3)$
$(1,2)\ (2,1)$
$(2,3)\ (3,2)$
$(3,1)\ (1,3)\}$

$$\text{Now, if } A\times A = \overbrace{\{\underbrace{(1,1)\ (2,2) \ldots (n,n)}_{\text{'n' elements}}\ \underbrace{(1,2)\ (2,1)\ (3,1)\ (1,3) \ldots}_{n^2-n \text{ elements}}\}}^{n^2 \text{ elements}}$$

'n' elements → choose All

n^2-n elements → choose $\frac{n^2-n}{2}$ ele

∴ The max cardinality of Antisymetric Relation $= \left(n + \frac{n^2-n}{2}\right)$

18. NUMBER OF ANTISYMETRIC RELATIONS.

$|A| = n \quad |A \times A| = n^2$

$A = \{1,2,3\}$ $\quad A \times A = \{ (1,1)\ (2,2)\ (3,3)$ — $2 \times 2 \times 2$

$(1,2)\ (2,1)\ (2,3)$ — 3

$(3,2)\ (1,3)\ (3,1) \}.$ — 3

$= 2^3 \times 3^3$ Relations

$= 2^{\text{diagnol ele}} \times 3^{\text{Nondiag pairs}}$

case 1: Include (1,2)

2: Include (2,1)

3: Include None

$\therefore$ $\boxed{\text{No. of Antisymetric Relations possible on A where } |A| = n \text{ is } = \left(2^n \times 3^{\frac{n^2-n}{2}}\right)}$

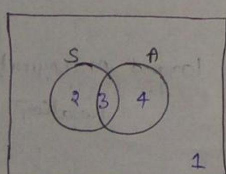
$\Downarrow$

$A \times A = \{ \underbrace{(1,1)\ (2,2)\ (3,3)\ \ldots\ (n,n)}_{n}\ \boxed{(1,2)\ (2,1)}\ \boxed{(3,2)\ (2,3)}\ \ldots \}$

$\boxed{(1,2)\ (2,1)} \to 3 \qquad \boxed{(3,2)\ (2,3)} \to 3 \ldots \ \frac{n^2-n}{2}$ times

$\Rightarrow 2^n \times 3^{\frac{n^2-n}{2}}$ Relations

19. RELATION BETWEEN SYMETRIC AND ANTISYMETRIC RELATIONS



$|U| = 2^{n^2}$

$n(S) = 2^{n(n+1)/2}$.

$n(A) = 2^n \times 3^{\frac{n^2-n}{2}}$

$R_1 = \{(1,2)\ (2,1)\ (2,3)\} \to$ Antisymetric ✗ Symetric ✗

$R_2 = \{(1,2)\ (2,1)\} \to$ S ✓ AS ✗

$R_3 = \{\ \} \to$ AS ✓ S ✓

$R_4 = \{(1,1)\} \to$ AS ✓ S ✓

$R_5 = \{(2,1)\} \to$ AS ✓ S ✗

$A \times A = \{ \underbrace{(1,1)\ (2,2)\ \ldots\ (n,n)}_{\text{Choose All}}\ (1,2)\ (2,1)\ \ldots \}.$

$= 2^n$

$\therefore$ $\boxed{\text{The no. of Relations that will be Both symetric and Anti symetric } = 2^n}$ $\to$ only diagonal elements

(6) Now, the no. of Relations that are either Symetric / Antisymetric are

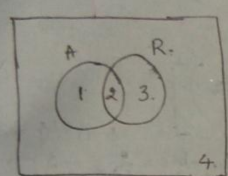
$$n(S \cup A) = n(S) + n(A) - n(S \cap A) \quad (A)$$

$$\boxed{n(S \cup A) = \left(2^{\frac{n(n+1)}{2}}\right) + \left(2^{n} \times 3^{(n^2-n)/2}\right) - \left(2^{n}\right)}$$

Similarly $n(S-A)$ and $n(A-S)$ can be found

$$\boxed{\begin{aligned} n(S-A) &= n(S) - n(S \cap A) \\ n(A-S) &= n(A) - n(S \cap A) \end{aligned}}$$

20. RELATION BETWEEN REFLEXIVE AND ANTISYMETRIC RELATIONS



$R_1 = \{(1,2)\}$

$R_2 = \{(1,1)\ (2,2)\ ---\ (n,n)\}$

$R_3 = \{(1,1)\ (2,2)\ --\ (n,n)\ (1,2)(2,1)\}$.

$R_4 = \{(2,1)\ (1,2)\}$

$n(A) = 2^{n}\ 3^{n(n-1)/2}$

$n(R) = 2^{n(n-1)}$

$n(u) = 2^{n^2}$

$A \times A = \{\underbrace{(1,1)\ (2,2)\ ---\ (n,n)}_{1}\ \boxed{(1,2)(2,1)}\ ---\ \}$.

$\times 3 \times 3\ ---$

Both Reflexive & Antisymetric $= 1 \times 3^{n(n-1)/2} = \boxed{3^{n(n-1)/2} = n(A \cap R)}$

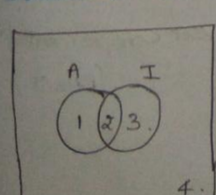
$n(A \cup R) = n(A) + n(R) - n(A \cap R) =$ Both Antisymetric and Reflexive

$n(A-R) = n(A) - n(A \cap R). =$ Antisymetric but not Reflexive

$n(R-A) = n(R) - n(A \cap R) =$ Reflexive but not Antisymetric

$n(\overline{A \cup R}) = n(U) - n(R \cup A) =$ Not Reflexive and not Antisymetric

21. RELATION BETWEEN IRREFLEXIVE AND ANTISYMETRIC RELATIONS



$R_1 = \{(1,1)\} \Rightarrow A✓\quad IR\times$

$R_2 = \{(1,2)\} \Rightarrow A✓\quad IR✓$

$R_3 = \{(1,2)(2,1)\} \Rightarrow A\times\quad IR✓$

$R_4 = \{(1,1)(1,2)(2,1)\}.\ A\times\quad IR\times$

$n(U) = 2^{n^2}$

$n(A) = 2^{n}\ 3^{n(n-1)/2}$

$n(I) = 2^{n(n-1)}$.

$A \times A = \{(1,1)\ (2,2) \cdots (n,n)\ (1,2)\ (2,1)\ (2,3)\ (3,2) \cdots\}$.

The Relations that are Both Irreflexive and Antisymetric

$A \times A = \{\underbrace{(1,1)\ (2,2) \cdots (n,n)}\ \underbrace{\boxed{(1,2)\ (2,1)}\ \boxed{(2,3)\ (3,2)} \cdots}\}$.

If i Include this they become reflexive so dont Include them

3choices 3 choices

$3^{(n^2-n)/2}$

$\therefore$ $\boxed{\text{The no. of Relations that are Both Antisymetric and Irreflexive is } = 3^{(n^2-n)/2}}$

Now, $n(A-I) = n(A) - n(A \cap I) = \left[\left(2^n 3^{n(n-1)/2} - 3^{(n^2-n)/2}\right)\right]$

$n(I-A) = n(I) - n(A \cap I)$ = Irreflexive but not Antisymetric $= 2^{n(n-1)} - 3^{(n^2-n)/2}$

$n(A \cup I) = n(A) + n(I) - n(A \cap I)$ = No. of Antisymetric but not Reflexive $= \left(2^n 3^{n(n-1)/2}\right) + 2^{n^2-n} - 3^{(n^2-n)/2}$

$n(\overline{A \cup I}) = n(U) - n(A \cup I)$

$2^{n^2} - \left(3^{(n^2-n)/2}\right)$

22. ANTISYMETRIC PROPERITIES.

State TRUE / FALSE

a) Every subset of Anti symetric relation is Antisymetric (TRUE)

b) " Superset " " " " " " " (FALSE)

c) Anti symetric relations are closed under set union. (FALSE)

d) " " " " " " Intersection (TRUE).

e) " " " " " " difference (TRUE)

f) " " " " " " Set Complementation - (FALSE)

23. EXAMPLES ON ANTI SYMETRIC RELATION

1) The Relation '≤' is Antisymetric on any set of Real numbers

2) The Relation '<' is Antisymetric on any set of Real numbers

3) The Relation "is a divisor" of denoted as '/' is an antisymetric on any set of +ve Real numbers.

4) The Relation '⊆' (set inclusion) is Anti symetric on any collection of sets

1) $x \leq y$ then (y,x) will not be present and ($y \leq x$ is false) ∴ The relation is Antisymetric

2) $x < y$ the ($y < x$) is false, so (y,x) will not be present in Relation therefore the relation is Anti symetric.

3) x/y (i mean if y is divisible by x)
↳ 'x' is divisor of 'y' then 'y' wont be divisor of x)
$2/4$ (4 is ÷ble by 2) then $(4/2)$ (2 wont be ÷ble by 5).

4) Antisymetric.

24. ASYMETRIC RELATIONS

A Relation 'R' on set 'A' is said to be Assymetric if $(x R y)$ then $(y \not R x)$ $\forall x,y \in A$

$A = \{1,2,3\}$

$R_1 = \{(1,2)\}$ Assymetric Relation (because (x,y) is present but (y,x) is not present).

$R_2 = \{(1,2)\ (2,2)\}$ Not Assymetric (because we included x,x)
Anti symetric (✓) because (x,y) is present and (y,x) is not present and $(2,2)$ is allowed case

Note: Diagnol elements they can be present in Antisymetric but not in Asymetric

$R_3 = \{\ \}$ Asymetric, Antisymetric, symmetric

$\boxed{\text{The min. cardinality of smallest Asymetric Relation = '0'.}}$

$R_4 = \{(1,2)(2,1)\}$ → Antisymmetric ✗ Asymetric ✗ Symetric ✓

$A \times A = \{(1,1)(2,2)(3,3)(1,3)(3,1)$
$(1,2)(2,1)$
$(2,3)(3,2)\}$.

$R_5 = \{(1,3)\ (1,2)\ (2,3)\}$ — This is the largest asymetric Relation.

Now, $A \times A = \{\underbrace{(1,1)\ (2,2) \cdots (n,n)}_{\text{They violate the Assymetric property}}\ \boxed{(1,2)\ (2,1)}\ \boxed{(3,2)\ (2,3)} \cdots\}$

Include one of $(1,2)$ & $(2,1)$

∴ $\boxed{\text{The cardinality of the largest Asymetric Relation that is possible with a set with 'n' elements is } = \left(\frac{n^2-n}{2}\right)}$

25. NO. OF ASYMERIC RELATIONS

$|A| = n \quad |A \times A| = n \times n$

Now, $|A \times A| = \{\underbrace{(1,1)\ (2,2) \cdots (n,n)}_{\text{should not Include it}}\ \boxed{(1,2)\ (2,1)}\ \boxed{(3,2)\ (2,3)}\ \boxed{(1,3)\ (3,1)} \cdots\}$

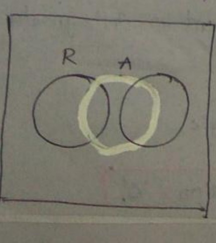
$\downarrow$ $3C1$ $\quad 3C1 \quad 3C1$

$= 3 \times 3 \times 3 \cdots \left(\frac{n^2-n}{2}\right)$ times.

∴ $\boxed{\text{The no. of Assymetric Relations} = 3^{\left(\frac{n^2-n}{2}\right)}}$

26. REFLEXIVE AND ASSYMETRIC RELATIONS

→ If a Relation is Reflexive then it cannot be Assymetric



$n(A) = 3^{\frac{n(n-1)}{2}}$

$n(R) = 2^{n(n-1)}$

$n(A \cap R) = 0$

$n(A - R) = n(A)$

$n(R - A) = n(R)$

$n(\overline{R \cup A}) = n(U) - n(R \cup A)$

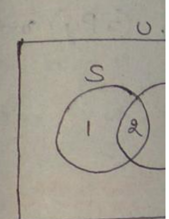
$n(A \cup R) = n(A) \cup n(R) = n(A) + n(R)$.

27. IRREFLE…

→ Every Assy…

→ Every Irre…

28. SYMETR…



Now, $n(S) \leq$ …

$n(S \cup A)$

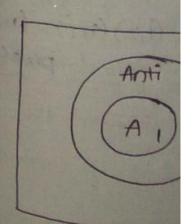
$n(\overline{S \cup A})$

$n(S - A)$

$n(A - S)$

29. ANTI SYMM…

Every Asymet…

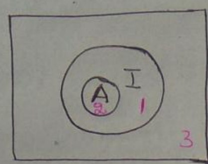


27. IRREFLEXIVE AND ASYMETRIC RELATIONS

(21)

⇒ Every Assymetric Relation is Irreflexive but

⇒ Every Irreflexive Relation is not Assymetric



$n(I) = 2^{n(n-1)}$

$n(A) = 3^{n(n-1)/2}$

$n(I \cup A) = n(I)$

$n(I \cap A) = n(A)$

$n(I - A) = n(I) - n(A)$

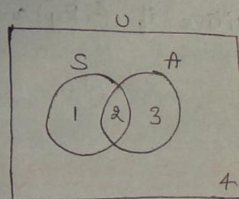
$n(A - I) = 0.$

$n(\overline{I \cup A}) =$

$= n(U) - n(I \cup A)$

$= n(U) - n(I)$

28. SYMETRIC AND ASYMETRIC RELATIONS



$R = \{(1,1)\}$ S ✓ AS ✗

$R_2 = \{\ \}$ S ✓ AS ✓

$R_3 = \{(1,2)\}$ S ✗ AS ✓

$R_4 = \{(1,1)(1,2)\}$ S ✗ AS ✗

$n(S) = 2^{\frac{n(n-1)}{2}}$

$n(A) = 3^{\frac{n(n-1)}{2}}$

$n(S \cap A) = 1$

Now, $n(S) \in n(S) - n(A \cap S)$ $n(S \cap A) = 1$

$n(S \cup A) = n(S) + n(A) - n(S \cap A)$

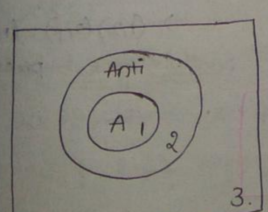
$n(\overline{S \cup A}) = n(U) - n(S \cup A)$

$n(S - A) = \{n(S) - n(S \cap A)\}$

$n(A - S) = \{n(A) - n(S \cap A)\}$

29. ANTISYMETRIC AND ASYMETRIC RELATIONS.

Every Asymetric Relation is Antisymetric



$R_1 = \{(1,1)(1,2)\}$ Antisymetric ✓ ASymetric ✗

$R_2 = \{(1,2)(2,1)\}$ Anti Symetric ✗ Asymetric ✗

$R_3 = \{(1,2)\}$ Anti Symetric ✓ Asymetric ✓

Now, $= n(A \cup Anti) = n(Anti)$

$= n(\overline{A \cup Anti}) = n(U) - n(Anti)$

$= n(Anti - Asy) = n(Anti) - n(Asy).$

$= n(Asy) - Anti) = 0.$

30. PROPERITIES OF ASYMETRIC RELATIONS

Assymetric Relations are closed under

a) subset operation (TRUE).

b) Superset operation (FALSE)

c) union " " (FALSE)

d) Intersection " (TRUE)

e) Set difference (TRUE)

f) Complementation (FALSE)

31. TRASITIVE RELATIONS.

A Relation R on set A is said to be transitive if (xRy) and (yRz) then (xRz) $\forall x, y, z \in A$

$R_1 = \{\}$ – Transitive

$R_2 = \{(1,1)\}$ – Transitive

$R_3 = \{(a,b)\ (c,d)\} \rightarrow$ Transitive (we dont have $(x,y)(y,z)$ pair)

$R_4 = \{(x,y)\ (y,z)\} \rightarrow$ NOT Transitive $[(x,z)$ is absent$]$

$R_5 = \{(x,y)\ (y,z)\ (x,z)\}$ = Transitive

$R_6 = \{(1,2)\ (2,2)\}$ – Transitive

$R_7 = \{(1,2)\ (2,1)\ (1,1)\}$ – Transitive $\Rightarrow (2,1)(1,1)$
$(2,1)$ is present
$\Rightarrow (1,2)(2,1) = (1,1)$ present

$R_8 = A \times A$ = Transitive

> The largest relation which is Transitive = $A \times A$
> The smallest relation which is Transitive = $\{\}$.

32. EQUIVAL

A Relation R

if R is 1)

Ex: $A = \{a, b, c$

$R_1 = \{(a,a)(b,b)($

$R_2 = \{(a,a)\ (b,b)$

$R_5 = \{(a,a)\ (b,$

The smallest

$\Rightarrow$ co

$\Rightarrow$ The largest

and it is

33. EXAMPLES

Which of the

of all Real nu

a) $R_1 = \{(a,b)/ a-$

b) $R_2 = \{(a,b)/a$

~~c)~~ $R_3 = \{(a,b)/$

d) $R_4 = \{(a,b)/$

34. POSET

Partial Ordering

A relation R

order) if R

Partially Orderd

A set A with

Orderd set (po

32. EQUIVALENCE RELATION

(22)

- A Relation 'R' on a set 'A' is said to be Equivalence Relation on 'A' if 'R' is 1) Reflexive 2) Symmetric 3) Transitive

(23)

Ex: $A = \{a, b, c\}$

$R_1 = \{(a,a)(b,b)(c,c)\}$ $R_3 = \{(a,a)(b,b)(c,c)(b,c)(c,b)\}$

$R_2 = \{(a,a)(b,b)(c,c)(a,b)(b,a)\}$ $R_4 = \{(a,a)(b,b)(c,c)(a,c),(c,a)\}$.

$R_5 = \{(a,a)(b,b)(c,c)(a,b)(b,c)(a,c)(c,a)(c,b)(b,a)\}$.

→ The smallest Equivalence set/Relation on set A contains 'n' elements. ⇒ contains only diagonal elements.

⇒ The largest Equivalence Relation on set A contains 'n^2' elements. and it is '$A \times A$'.

33. EXAMPLES OF EQUIVALENCE RELATIONS.

Which of the following is not an Equivalence relation on a set of all Real numbers?

a) $R_1 = \{(a,b) / a-b \text{ is a integer}\}$ — R ✓ All diagonal ele diff = 0 (integer); S ✓ If (a-b) is integer (b-a) is also integer; T ✓

b) $R_2 = \{(a,b) / a-b \text{ is } \div\text{le by } 5\}$ — R ✓ S ✓ T ✓

c) $R_3 = \{(a,b) / a-b \text{ is odd no}\}$ → Not Reflexive (diagonal ele diff = 0 = even no.

d) $R_4 = \{(a,b) / a-b \text{ is an even no.}\}$ → R ✓ (All diagonal ele are present); S (symmetric); T (Transitive) } = Equivalence Relation.

34. POSET

Partial Ordering Relation:

A relation 'R' on set A is said to be partial ordering relation (partial order) if 'R' is Reflexive, Antisymmetric, and Transitive

Partially Orderd Set

A set 'A' with a partial order 'R' defined on 'A' is called partially Orderd set (poset) and it is denoted by [A ; R]

$A = \{1,2,3\}$

$R_1 = \{(1,1)(2,2)(3,3)\}$ – Reflexive ✓, Transitive ✓, Symetric ✓, Antisymetric ✓

$\Rightarrow$ The Relation is Equivalence and partial order Relation.

$\Rightarrow$ This is the smallest Relation which is both partial ordering and Equivalence Relation.

$R_2 = \{(1,1)(2,2)\} \rightarrow$ cannot be Equivalent and partial ordering (Because this doesnot contain (3,3))

$R_3 = \{(1,1)(2,2)(3,3)(1,2)\} \Rightarrow$ Not Equivalent Relation

Reflexive ✓ Antisymetric ✓ Transitive ✓ } partial ordering Relation.

Now, let us consider set of all Real numbers 'R'. Now, let the Relation be "≤" then 'R' is Reflexive, Antisymetric, Transitive. Therefore $[R; \leq]$ is called the partial order set / poset.

This Relation (≤) is the partial orderd set on Relation 'R'.

Now, $[S; \subseteq]$ is also a poset —
$[R; /]$ is also a poset —
} Necessary condition is the Relation should be Reflexive, Anti symetric, Transitive.

35. TOS

Totally orderd set (Linearly orderd set or chain)

A poset $[A;R]$ is called a "Totally Orderd Set" if every pair of elements in 'A' are comparable, i.e aRb or bRa $\forall a,b \in A$

Ex: Find whether the following are totally orderd sets or not

1) If 'A' is any set of real no's then poset $[A, \leq]$ is TOS
2) If $A = \{1,2,3,4 \text{---} 10\}$ then the poset $[A, \leq]$ is a TOS.
3) If $A = \{1,2,6,30,60,300\}$ then $[A; /]$ is TOS [2 ÷ 6, 6 ÷ 300, 60 --].
4) If $S = \{\phi, \{a\}, \{b\}, \{a,b\}\}$ then $[S, \subseteq]$ is Not TOS.
5) If $S = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}\}$ then $[S, \subseteq]$ is TOS.

1) Now, if i take ...
them, { if i t...
 .if i ...

4) $\{a\} \notin \{b\}$ ...

5) $\{a\} \subseteq \{a,b\}$.
$\{a,b\} \subseteq \{a,b,c\}$.

36. GATE QUEST...

Let $A = \{a,b,c\}$ wh...

a) $R_1 = \{(a,a)(c,c)\}$
b) $R_2 = \{(a,b),(b,a)(a$...
c) $R_3 = \{(a,b)(b,a)(c$...
d) $R_4 = \{(a,b)(b,c)(c,$...

37. GATE QUESTIO...

Let $A = \{a,b,c,d\}$ ...
$(b,a)(b,b)(b,c)(b,d)$...

a) R is Equivalence
b) R is irreflexive
c) R is symetric
d) R is transitive

38. GATE. QUESTION 3...

Let A = set of all ...
i.e $aRb \Leftrightarrow b=a^k$

a) R is Equivalence ...
b) 'R' is partial order...
c) 'R' is reflexive and ...
d) 'R' is total order.

$R = \{(2,2)(2,4)(2,8)(2,$...
$(1,1)(3,3)(3,9)(3$...

3> Now, if i take any two Real no's, i can put the relation '$\leq$' in btw them, $\left\{\begin{array}{l}\text{if i take 1,2 i can say } (1\leq 2)\\ \text{if i take 4,2 i can say } (2\leq 4)\end{array}\right\}$ $\therefore$ Every pair of Real no's is Comparable

$\Rightarrow$ This is "TOS."

4> $\{a\} \nsubseteq \{b\}$ $\therefore$ This is not a "Total orderd set."

5> $\{a\} \subseteq \{a,b\}$. $\phi \subseteq \{a\}, \{a,b\}, \{a,b,c\}$

$\{a,b\} \subseteq \{a,b,c\}$.

25

36. GATE QUESTION-1

Let $A=\{a,b,c\}$ which of the following is Not TRUE (FALSE)?

a) $R_1=\{(a,a)(c,c)\}$ is symetric, Antisymetric, Transitive (TRUE)

b) $R_2=\{(a,b),(b,a)(a,c)\}$ is symetric, Antisymetric (FALSE) ((c,a) is not present)

c) $R_3=\{(a,b)(b,a)(c,c)\}$ is symetric but not Antisymetric (TRUE)

d) $R_4=\{(a,b)(b,c)(c,c)\}$ is Antisymetric but not symetric. (TRUE)

37. GATE QUESTION 2

Let $A=\{a,b,c,d\}$ and a relation on set A is defined as $R=\{(a,a)(b,a)(b,b)(b,c)(b,d)(c,a)(c,b)(c,c)(c,d)\}$ which of the following is TRUE?

a) R is Equivalence Relation = NOT EQUIVALENCE (NOT REFLEXIVE (d,d) X).

b) R is irreflexive or Antisymetric relation FALSE (NOT IRREFLEXIVE)

c) R is symetric or Asymetric Relation ( )(FALSE) $\{(a,b)\}$ is absent

d) R is transitive. ~~(FALSE)~~ ~~(b,a)(a,a) ⇒ (b,a) (not present in R)~~ (TRUE)

38. GATE. QUESTION 3

Let A = set of all Real numbers, $R=\{(a,b)/b=a^k$ for some integer $k\}$

ie $aRb \Leftrightarrow b=a^k$

a) R is Equivalence Relation: R ✓ $(a,a) \Rightarrow a=a^1$; S ×; T $\}$ $\therefore$ Not Equivalent

b) 'R' is partial order ✓: R ✓, Anti ✓, T ✓ $\}$ $\therefore$ The Given Relation is partial order.

c) 'R' is reflexive and symetric but not Transitive

d) 'R' is total order.

$R=\{(2,2)(2,4)(2,8)(2,16)$ - - -
$(1,1)(3,3)(3,9)(3,27)$ - - - $\}$.

NOT Symetric = (8,2) is not present
$\Rightarrow 2=8^{1/3} \rightarrow$ but K is Integer

39. GATE QUESTION 4

Which of the following is Not TRUE (FALSE)?

a) ✗ If a Relation 'R' on set A is symetric and Transitive then 'R' is Reflexive

b) If a relation 'R' on set 'A' is irreflexive and transitive then 'R' is Antisymetric

c) ✗ If 'R' and 's' are Antisymetric on 'A' then $(R \cup S)$, $(R \cap S)$ are also Antisymetric

d) If R, S are Transitive then

a) A = {1,2,3}   A = {} → Not Reflexive, but not (Symetric & Transitive).

R = {(1,1)} – Transitive, symetric, not Reflexive ⇒ op1 : FALSE.

b) R = TRUE . (C) FALSE   (D) TRUE

41. GATE QUESTION 6

The no. of equivalence relations on set {1,2,3,4} is

a) 15   b) 16   c) 24   d) 4

Just Remember if a set has

3 elements → No. of Equivalence Relations = 5

4 elements → No. of Equivalence Relations = 15

R = {

SUMMARY ON RELATIONS

1> **Reflexive Relation:** All the diagnol ele should definitely be present

2> **Irreflexive Relation:** No diagonal ele should be present if you find atleast one diagnol ele then it is NOT IRREFLEXIVE

3> **Symetric Relation:** If (x,y) is present then only check for (y,x) (All the symetric pairs need not be present).

{ } - symetric {Min cardinality = 0, Max cardinality = n^2 }.

4> **Anti symetric Relation:** Symetric pairs should not be present but diagnol pairs are allowed (Exception case)

{ } - Anti symetric {Min cardinality = 0. Max cardinality = $n + \frac{n^2-n}{2}$ }.

5> **Asymetric Relation:** → symetric pairs should not be present, No exception on diagnol elements also.

{ } - Asymetric {Min cardinality = 0, Max cardinality = $\frac{n^2-n}{2}$ }.

6> **Transitive Relation:** If (a,b) is present and (b,c) is present then only check for (a,c) if (a,c) is presents then Transitive else Not Transitive.

⇒ Iff, $(a,b)\ (b,c) \Rightarrow (a,c)$. | $R = \{abc\}$ | $R_1 = \{(ab)(bc)\}$ = Not Transitive

⇒ { } = Transitive {Min cardinality = 0. Max cardinality = $n^2 (A \times A)$ }.

7> **Equivalence Relation:** → Reflexive, → Symetric, → Transitive } (TRS).

8> **POSET** : Reflexive, Antisymetric, Transitive (RAT).

9> **TOS.** : Every pair should be comparable on the Relation given.

3. PARTIAL ORDERS AND LATTICES

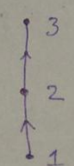
1. POSET DIAGRAM (HASSE DIAGRAM)

Let $[A:R]$ be a poset. The poset diagram is as follows

1) There is a vertex corresponding to each element of 'A'.
2) An edge between the elements 'a' and 'b' is not present in the diagram if there exists an element $x \in A$ such that (aRx) and (xRb)
3) An edge b/w the elements 'a' and 'b' is present iff aRb and there is no element $x \in A$ such that (aRx) and (xRb).

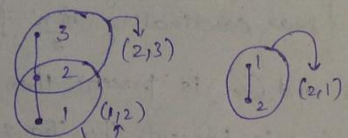
Ex: $A = (1,2,3)$

$R = \leq = \{(1,2)(1,3)(2,3)$
$(1,1)(2,2)(3,3)\}$.



(1,3) is not needed because (1,2)(2,3) ⇒ (1,3), anyway (1,2) (2,3) edges are present

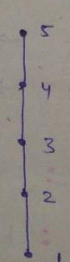
⇒ The general convention that we assume is bottom to top and we dont use arrows also.



2. EXAMPLES ON POSET DIAGRAMS

$A = \{1,2,3,4,5\}$, $R = \leq$ } poset = $[A; \leq]$

$R = \{(1,1)(2,2)(3,3)(4,4)(5,5)$
$(1,2)(1,3)(1,4)(1,5)(2,3)$
$(2,4)(2,5)(3,4)(3,5)$
$(4,5)\}$

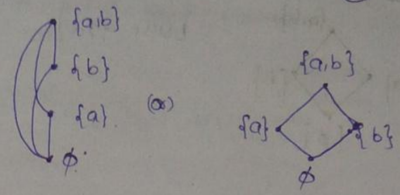


⇒ Here the diagram looks like chain

⇒ The relation in TOR (Total order Relation)

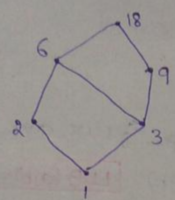
② $S = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

poset: $[S; \subseteq]$



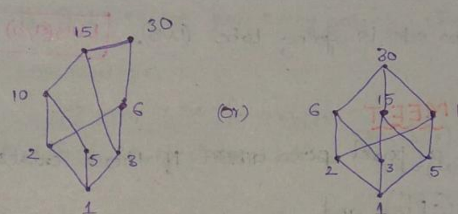
③ $A = \{1,2,3,9,6,18\}$.

poset: $[A, /]$



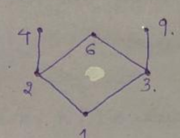
④ $A = \{1,2,3,5,6,10,15,30\}$.

poset: $[A, /]$



⑤ $A = \{1,2,3,4,6,9\}$.

poset $[A, /]$

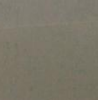


3. LUB

Least upper Bound (LUB or Join or Supremum)

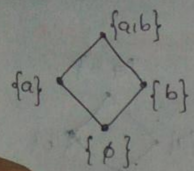
Let $[A;R]$ be a poset. for $a,b \in A$, if there exists an element $c \in A$ such that i) aRc and bRc,

ii) if there exists any other element 'd' such that (aRd) and (bRd) then (cRd), then 'c' is the LUB of 'a' and 'b'.

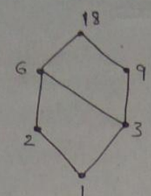


Ex :

$[D_{18}, /]$

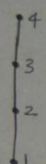


LUB = {a,b}.



LUB = {18}

⇒ In case of divisor operation $\boxed{LUB(a,b) = lcm(a,b)}$.



[A; ≤]

In case of ≤ Relationship if we are comparing (a,b) then the LUB will be max(a,b). $\boxed{LUB(a,b) = max(a,b)}$

⇒ In this diagram LUB = {4}.

⇒ In case of set Inclusion Relation (subset or equalto Relation) then union of two sets is going tobe (LUB). $\boxed{LUB(a,b) = \{a\} \cup \{b\}}$

4. GLB OR MEET

Let [A; R] be a poset for a,b∈A if there exists an element c, such that

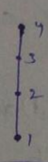
i) (c R a) and (c R b) and

ii) If there exists any other element 'd' such that (d R a) and (d R b) then (d R c) then 'c' is called the GLB of 'a' and 'b'.

(c, a)
(c, b)

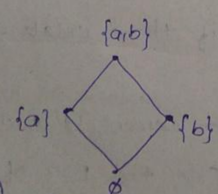
(d, a)
(d, b)

(c, d) → c is called the Greatest lower Bound.



$\boxed{GLB(a,b) = min(a,b)}$

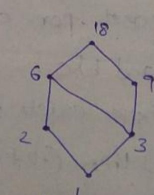
≤ Relationship



$\boxed{GLB(a,b) = a \cap b}$

Set Inclusion

Subset / Equalto



$\boxed{GLB(a,b) = GCD(a,b)}$

STANDARD EXAMPLES.

1) $[A: \leq]$ LUB $(a,b) = \max(a,b)$ A = set of Real numbers
GLB $(a,b) = \min(a,b)$

2) $[A; /]$ LUB$(a,b) =$ Lcm(a,b) A = set of Real no's.
GLB$(a,b) =$ GCD(a,b)

3) $[S; \subseteq]$ LUB = union . S = set of all sets.
GLB = Intersection.

for a given pair of elements the LUB, GLB may or may not exists.

6. LATTICE

<u>Join semi lattice</u>: If LUB exists for every pair of elements in poset

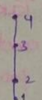
<u>Meet semi lattice</u> If GLB exists for every pair of elements in poset.

<u>lattice</u>: If Both LUB and GLB " " " " " " " "

Ex: $A = \{1,2,3 --- 10\}$ $[A, /]$ is meet semi lattice ↳ GLB exists for every pair. [For (3,4) there is no LUB = (Lcm(3,4) = 12) and 12 is not in set, so not LUB ⇒ Not JSL)].

$S = \{\{a\}, \{b\}, \{a,b\}\}$ then $[S, \subseteq]$ is join semi lattice [GLB does not exist for $\{a\}\{b\}$ (Intersection of $\{a\}\{b\} = \phi$ (not in set).

$A = \{1,2,3,4\}$ $[A, \leq]$ is a lattice.



$\boxed{\begin{array}{l} \text{GLB} = (\max(a,b)) \\ \text{LUB} = \min(a,b) \end{array}}$

∴ A TOS (Total order set) is always a lattice.

7. LATTICE EXAMPLES.

→ If 'A' is set of all +ve integers, then poset $[A; /]$ is a lattice
(a,b) ⇒ Lcm = +ve no
(a,b) ⇒ GCD = +ve no
∴ LUB, GLB exists

→ If 'n' is a positive integer then D_n = set of all +ve divisors of 'n'.

Ex: $D_6 = \{1,2,3,6\}$.

$D_{12} = \{1,2,3,4,6,12\}$

$D_{30} = \{1,2,3,5,6,10,15,30\}$.

Every $[D_n, /]$ is a lattice.

→ If p(A) denotes powerset of A then $[p(A); \subseteq]$ is a lattice
A = {a,b}
p(A) = {φ, {a}, {b}, {a,b}}.

8. PROPERITIES OF LATTICE

The following properity holds good in a lattice for any 3 ele $a,b,c \in A$

i) Commutative law: $a \vee b = b \vee a$
$a \wedge b = b \wedge a$

ii) Associative law:
$(a \vee b) \vee c = a \vee (b \vee c)$
$(a \wedge b) \wedge c = a \wedge (b \wedge c)$

iii) Idempotent law: $a \vee a = a$
$a \wedge a = a$

(iv) Absorption: $a \vee (a \wedge b) = a$
$a \wedge (a \vee b) = a$

v) Note: In a lattice $(a \vee b) = b$ iff $(a \wedge b) = a, \forall a,b \in L$ $\left\{ \begin{array}{l} \vee = LUB \\ \wedge = GLB \end{array} \right\}$

9. DISTRIBUTIVE LATTICE AND SUBLATTICE

The lattice on which the distributive properity holds is called Distributive lattice, and the distributive properities are

i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ $\left\{ \begin{array}{l} \vee = LUB \\ \wedge = GLB \end{array} \right\}$.

Sublattice:

Let L be a lattice $[L, \wedge, \vee]$. A subset 'M' of 'L' is called a sublattice of 'L' iff

i) M is a lattice i.e $[M, \vee, \wedge]$

ii) For any pair of elements $a,b \in M$ the LUB and GLB are same in M and L

> To find whether a lattice is distributive there are 2 ways
> 1) Take all possible Triplets and check distributive properities (Headache)
> 2) Use the process of / Concept of sublattices

$A \rightarrow A \times A \rightarrow R \rightarrow R,A,T \rightarrow poset \rightarrow [A; R] \rightarrow lattice \xrightarrow{D} Distributive\ lattice$
$\downarrow$
M (sublattice)

10. BOUNDED LATTICE

→ Let 'L' be a lattice with respect to R, if there exists an element $I \in L$ such that $(aRI)\ \forall a \in L$, then I is called "Upper bound of Lattice L".

→ similarly if there exists an element $O \in L$, such that $(ORa)\ \forall a \in L$, then 'O' is called "Lower bound of Lattice L"

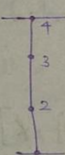
→ In a lattice if upper bound and Lower bound exists then it is called Bounded lattice.

LUB, GLB } → Calculated on pair of elements.

Upper Bound, Lower Bound } → Calculated on entire lattice not on pair of elements

⇒ Every finite lattice is Bounded.

$\{\{1,2,3,4\}, \leq\}$

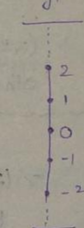


upper bound (4 is related to all other ele like (4,4) (3,4) (2,4) (1,4)

1 (lower bound because 1 is related to all other elements (1,1) (1,2) (1,3) (1,4)

⇒ For a lattice there may/may not be upper bound and lower bound.

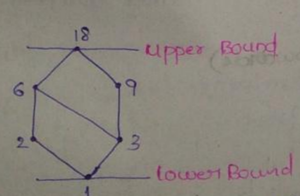
$[I, \leq]$ ↓ set of Integers.



No upper bound and lower bound.

⇒ $[D_{18}; /]$

$D_{18} = \{1, 2, 3, 6, 9, 18\}$.



(1,1) (1,2) (1,3) (1,6) (1,9) (1,18)

11. PROPERITIES OF BOUNDED LATTICE

In a Bounded lattice, the following properities holds good. (34)

1> LUB of a and I i.e $a \vee I = I$ $\{(a, I)\}$
2> GLB of a and I i.e $a \wedge I = a$ $\{(a, I)\}$
3> LUB of a and 0 i.e $a \vee 0 = a$ $\{(0, a)\}$
4> GLB of 'a' and '0' i.e $a \wedge 0 = a$ $\{(0, a)\}$.

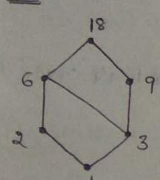
$\left\{ \begin{array}{l} I - \text{Upperbound of lattice} \\ 0 = \text{Lower bound of lattice} \end{array} \right.$

12. COMPLIMENT OF AN ELEMENT

Let 'L' be an bounded lattice, for any element $a \in L$, if there exists an element $b \in L$ such that $(a \vee b) = I$ and $(a \wedge b) = 0$, then 'b' is called 'Compliment of a' written as $\bar{a}$. and 'a', 'b' are compliments of each other.

⇒ "Compliment" is only possible for "Bounded lattice."

Ex:

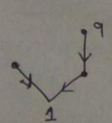


(2,9) are compliment to each other

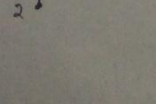
⇒ $(2 \vee 9)$ = Join operation = 18 (UB)
$(2 \wedge 9)$ = meet operation = 1 (LB)

$2 = \bar{9}$ or $9 = \bar{2}$

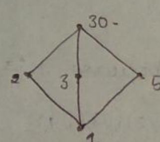
(2,9) are compliment



⇒ meet operation (come downwards).



= Join operation (go upwards)



$[\{1, 2, 3, 5, 30\}; /]$

$(2 \vee 3) = 30$ → upperbound (Join operation)

$(2 \wedge 3) = 1$ (Meet operation) → lowerbound

⇒ (2,3) are compliment of each other ⇒ $\bar{2} = 3$ or $\bar{3} = 2$

⇒ (2,3) are not Compliment to each other because

$(2 \vee 3) = 6$ (Not UB) → ✗
$(2 \wedge 3) = 1$ (LB)

we should get both UB & LB.

13. COMPLEM...
⇒ If every ... lattice.
⇒ In a complem...
⇒ In a Distrib...
i.e each e...
$[\{1, 2, 3, 6\}; /$...
$[\{1, 2, 3, 5, 30\}; /$...
⇒ In a Distrib...

14. BOOLEAN A...
Boolean Algebra
A lattice 'L' is ...
complemented ...
⇒ In Boolean Al...
unique comple...

15. MAXIMAL ...
Maximal element: ...
other element, th...
Minimal element: ...
it is called mini...
Ex: $A = \{a, b\}$.
$[P(A); \subseteq]$
$\{a\}$...

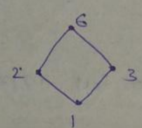
13. COMPLEMENTED LATTICE

⇒ If every element of a lattice has compliment then it is called Complimented lattice.

⇒ In a complemented lattice, each element has atleast one Compliment

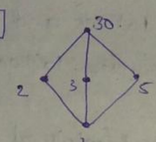
⇒ In a Distributive lattice, compliment of an element if exists, is unique i,e each element has atmost one complement.

[{1,2,3,6} : /]



→ Complimented lattice every ele has Compliment.

[{1,2,3,5,30}; /]



⇒ Compliment lattice

⇒ In a Distributive lattice each ele has (0 compliment & 1 compliment).

14. BOOLEAN ALGEBRA

Boolean Algebra

A lattice 'L' is called Boolean Algebra if it is distributive and complemented

⇒ In Boolean Algebra every element has atmost one Complement. i,e unique complement.

15. MAXIMAL AND MINIMAL ELEMENTS.

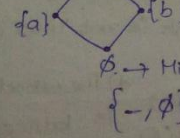
Maximal element : If in a poset, an element is not related to any other element, then it is called maximal element.

Minimal element: If in a poset, an ele is related to an element, then it is called minimal element.

(ϕ, {a}) → ϕ is Related to {a}, Not {a} is related to ϕ.

Ex: A = {a,b}.

[P(A); ⊆]

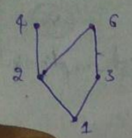


{a,b} → maximal ele

bcoz {{a,b}, —} {(a,b)} is not related to any other element

ϕ → Minimal elements

{—, ϕ} → No ele is related to 'ϕ' so ϕ is the minimal ele.



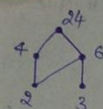
[{1,2,3,4,6}; /]

Maximal elements = 4,6.

Minimal elements = 1

This is not a lattice because {4,6} donot have LUB. and it is Meet semi lattice.

(36)



Maximal elements = 24.

Minimal elements = 2,3.

Join semi lattice

* The upper Bound and lower bound are unique but maximal/minimal ele need not be unique.

⇒ Maximal, Minimal are applicable to both latices and posets but UB,LB can be /must be applied only for Bounded lattices.

⇒ In a poset if you have more than one maximal/minimal ele then it wont be lattice.

16. EXAMPLE 1

The poset [{2,3,4,9,12,18}; /] is

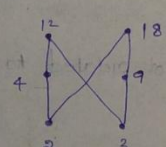
a) Join semi latice but not meet semilattice

b) Meet semi latice but not Join SL

c) A lattice

d) Neither join nor meet semi lattice.

⇒



Maximal ele = 12,18

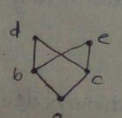
Minimal ele = 2,3

∴ This is not lattice

And there is no GLB for 2,3 and LUB for 12,18 ∴ They arenot Join and meet SL

17. EXAMPLE-2

The poset diagram of a poset p = {a,b,c,d,e} is shown below.



which of the following statements is not TRUE (FALSE)?

a) 'p' is not lattice (True)

b) The subset {abcd} is lattice (T)

c) The subset {b,c,d,e} is lattice (F)

d) The subset {a,b,c,e} of 'p' is lattice (T)

a) TRUE becua

b) TRUE all

c) FALSE {d,e

18. EXAMPLE-

The Hasse dia

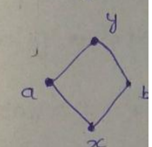
which of the foll

a) {x,a,b,y}

c) {x,a,e,y}

(e) {x,c,d,y}

a) {x,a,b,y}



⇒ lattice becuas every pair of elem LUB, GLB Exists.

⇒ For sublattice f LUB of every pa the options, Here, GLB and LUB a but for original di LUB (a,b) = C (≠ Cannot be the su

a) TRUE becuase (d,e) donot have least upper bound.

b) TRUE all pairs have LUB, GLB

c) FALSE [(d,e) has no LUB and (b,c) donot have GLB)

18. EXAMPLE-3

The Hasse diagram of a lattice $L = \{x, a, b, c, d, e, y\}$ is shown below which of the following subsets of 'L' are sublattices of L?

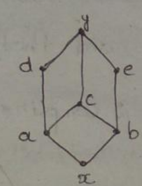
a) $\{x, a, b, y\}$ (b) $\{x, a, c, y\}$

c) $\{x, a, e, y\}$ (d) $\{x, d, e, y\}$

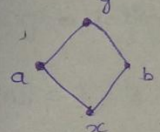
(e) $\{x, c, d, y\}$.

GLB = కింద

LUB = మీద



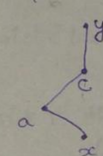
a) $\{x, a, b, y\}$



⇒ lattice becuase for every pair of elements both LUB, GLB Exists.

⇒ For sublattice find the GLB, LUB of every pair of ele in the options, Here, for (a,b) the GLB and LUB are x, y respectively but in original diagram the LUB (a,b) = C ($\neq y$) ∴ This Cannot be the sublattice of 'L'.

(b) $\{x, a, c, y\}$.



⇒ Lattice bcoz LUB, GLB exists for every pair

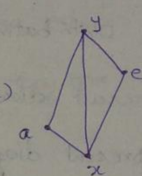
⇒ For (a,x) = GLB = x
LUB = a.

For (a,c) = GLB = c
LUB = a

For (c,y) = GLB = c
LUB = y

These are same in Both diagrams ∴ It is sublattice

(c)

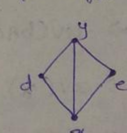


⇒ lattice For (a,e) GLB = x, LUB = y } Same in original diagram

For (a,y) GLB = a
LUB = y.

∴ This is the sublattice

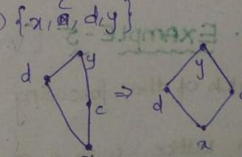
(d) $\{x, d, e, y\}$.



⇒ lattice

⇒ sublattice also.

(e) $\{x, c, d, y\}$



⇒ GLB(d,c) = x
GLB(d,c) = a (In original diagram)

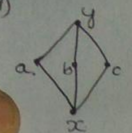
∴ Not a sublattice

19. EXAMPLE-4 (V. Imp Question)

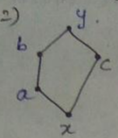
Which of the following latices is not distributive

(38)

1)



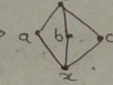
2)



⇒ If a lattice is Distributive then we should have atmost one complement for each element so in the first diagram 'a' has two complements 'b' and 'c' and therefore it cannot be distributive

⇒ In the 2nd Graph 'c' has 2 compliments 'a' and 'b'. so the 2nd diagram is also not Distributive

let L_1^* represent diag 1 ⟶



GLB ⟵	$\wedge$ = see down
LUB. ⟵	$\vee$ = see up.

L_2^* represent diag 2.

⇒ In L_1^* ⇒ $a\vee(b\wedge c) \overset{?}{=} (a\vee b)\wedge(a\vee c)$

$\Rightarrow a\vee(x)$ | $(y)\wedge(y)$

$\Rightarrow a$ | $= y$

$\Rightarrow (a \neq y)$ ∴ The distributive property doesnot hold true

⇒ In L_2^* ⇒ $a\vee(b\wedge c) \overset{?}{=} (a\vee b)\wedge(a\vee c)$

$\Rightarrow a\vee(x)$ | $\Rightarrow (b)\wedge(y)$

$\Rightarrow a$ | $\Rightarrow b$

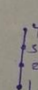
$(a \neq b) \Rightarrow$ The distributive property doesnot hold true for this lattice

20. EXAMPLE-5 (V. Imp Question)

which of the following statements are not true

a) A lattice with 4 or fewer elements is distributive (TRUE)

b) Every totally orderd set is a distributive lattice (TRUE) ⟶



c) Every sublattice of a distributive lattice is also distributive (TRUE)

No Isomorphic Structures exist similar to L_1^* or L_2^*.

d) Every distributive lattice is a bounded lattice

⇒ we have already seen L_1^*, L_2^* ~~as sublattice and they~~ are not distributive lattices. so If a lattice contains L_1^* or L_2^* as sublattice

---

then that

d). NO Relati

21. EXAmpl

Which of th

a) $[P(A), \subseteq]$

b) $[D_{81} ; /]$

c) $[R ; \leq]$ R

d) $[\{1, 2, 3, 5, 3$

⇒ To check wh

L_2^* are not

the given lat

a) $[P(A), \subseteq]$

union and

we know that

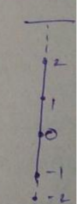
and Intersect

Distributive

b) $D_{81} = [\{1, 3, 9,$

For every val

c) $[R ; \leq]$



d) $[\{1, 2, 3, 5, 30\} ;$

~~$[D_{30} ; /]$~~

then that lattice won't be ~~sublattice~~ distributive lattice.

d) No Relation btn Distributive and Bounded lattices (FALSE)

21. EXAMPLE-6

Which of the following is not distributive lattice?

a) $[P(A), \subseteq]$ where $A = \{a, b, c, d\}$.

b) $[D_{81}; /]$

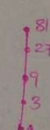
c) $[R; \leq]$ R is set of Real numbers

d) $[\{1, 2, 3, 5, 30\}; /]$

⇒ To check whether a lattice is distributive or not we check if L_1^* and L_2^* are ~~not~~ sublattices of given lattice, if they are ~~sublattices~~ then the given lattice is not distributive.

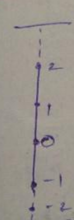
a) $[P(A), \subseteq]$ $A = \{a, b, c, d\}$ w.k.t the join of two elements is nothing but union and meet of two elements is nothing but Intersection and we know that on a set of all sets Union is distributive over Intersection and Intersection is distributive over union. Therefore the option a is Distributive

b) $D_{81} = [\{1, 3, 9, 27, 81\}; /]$



$\boxed{\text{For every value of 'n' } [D_n; /] \text{ is always Distributive}}$

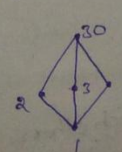
c) $[R; \leq]$



⇒ This is a Total order Relation and w.k.t every Total order Relation is $\boxed{\text{Distributive.}}$

d) $[\{1, 2, 3, 5, 30\}; /]$

~~$[D_{30}; /]$~~



= Isomorphic to L_1^* so this lattice is Not distributive. and (2,3)(2,5) are Compliment. '2' has two Compliments.

22. EXAMPLE-7

For the lattice $[D_{18}; /]$ which of the following is not TRUE.

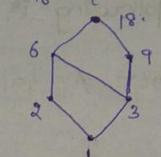
a) The compliment of $1 = 18$

b) The compliment of $2 = 9$

c) The compliment of $3 = 6$ $(3 = \bar{2})$

d) The compliment of '6' doesnot exists

$D_{18} = \{1, 2, 3, 6, 9, 18\}$



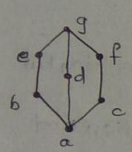
$(6 / 9)$

$(6 \vee 9) = 18$ (LUB)
$(6 \wedge 9) = 3$ (Not GLB) } 6,9 are not compliment

{GLB = 18 In the lattice}

23. EXAMPLE-8

For the lattice given below, how many complements does the ele 'e' have?



(e,a)
(e,b)
(e,c)
(e,d)
(e,f)
(e,g)
} These are the combinations possible

Now, The upper bound of lattice = g
lower bound of lattice = a

Join = $\wedge$ = ~~upper~~ lower bound
meet = $\vee$ = ~~lower~~ upper bound

Now, $(e \wedge a) = a$
$(e \vee a) = e$ } (e,a) Not Compliment

$(e \wedge b) = b$
$(e \vee b) = e$ } Not compliment.

$(e \wedge d) = a$ - LB of lattice
$(e \vee d) = g$ = UB of lattice } (e,d) are compliment.

$(e \wedge f) = a$
$(e \vee f) = g$ } (e,f) are compliment

Similarly (e,c) are compliment to each other.

4. GROUPS.

1. ALGEBRAIC STRUCTURES

A Non-empty set 'S' is called an Algebraic structure with respect to Binary operation $*$ if $(a*b) \in S$ $\forall$ $a,b \in S$ i,e $*$ is closure operation on 'S'

1) $S = \{1, -1\}$.

   $* \rightarrow \times$ (multiplication)

   $(S,*)$ is Algebric structure becuase $(1)\times(-1) = -1$ (present in 'S')

   $-1 \times -1 = 1$ $(\in S)$

   $1 \times 1 = 1$ $(\in S)$

   $\therefore$ 'S' is called the Algebraic Structure.

2) $S = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

   $* = \cup$ (union operation)

   Now $(S,*)$ is A.S. becuase

   $\phi \cup \{a\} = \{a\} \in S.$

   $\phi \cup \{b\} = \{b\} \in S.$

   $\{a\} \cup \{a\} = \{a\} \in S.$

   $\phi \cup \phi = \phi \in S.$

   $\{a,b\} \cup \{a\} = \{a,b\} \in S.$

   ⋮

3) $A = \{1,2,3\}$

   R = set of all Reflexive Relations $= \{(1,1)(2,2)(3,3)\}$.

   $(R,\cup) =$ A.S.

   $(R,\cap) =$ A.S.

   The set of all Reflexive Relations are closed under Union/Intersection and thefore set of all Reflexive Relations w.r.t to Intersection/Union is an "ABELIAN STRUCTURE"

4) $(R,+) \Rightarrow$ "ABELIAN STRUCTURE" (R = Real no's)

5) $(N,*) \Rightarrow$ "ABELIAN STRUCTURE" (N = Natural numbers)

6) $(S = \{1,2,3\}$ $\Rightarrow$ (NOT Algebraic STURCUTURE)

   $* \rightarrow$ multiplication

   Now, $2*3 = 6$ (Not in S). so

7) $[Z, /] \Rightarrow$ Not Algebraic Structure.

   $2/3 = 0.6 \neq$ Integer

2. SEMI GROUP

An Algebraic Structure $(S,*)$ is called a Semigroup if $(a*b)*c = a*(b*c)$ $\forall\ a,b,c \in S$ i,e $*$ is Associative on 'S'.

Ex:

Naturalno = Natural No.

1) $(N,+)$ = Algebraic Structure (✓) $\rightarrow$ $a+(b+c)$ $(a+b)+c = a+(b+c)$
2) $(N,*)$ = (Algebraic Structure (✓) $\rightarrow (a\times b)\times c = a\times(b\times c)$ ∴ SEMI GROUP
3) $(Z,-)$ = Z = {Integers set} AS(✓) $\leftrightarrow$ $(a-b)-c = a-(b-c)$ (F) ∴ NOT SEMI GROUP
   1 2 3   1 2 3
4) $(Q^*,+) = Q$ = {Rational No's} nothaving '0'. AS(✗) $a+(-a)=0$ $-4 \neq 2$
   Not Rational.
5) $(Q^*,*)$ = SemiGroup
6) $(P(A),\cup)$ = Semi Group
7) $(P(A),\cap)$ = Semi Group

3. MONOID

A semigroup $(S,*)$ is called Monoid if there exists an element $e \in S$ such that $(a*e) = (e*a) = a$ $\forall a \in S$. ($*$ = operation defined)

The element 'e' is called Identity element of 'S' w.r.t. $*$.

⇒ If a semi Group contains Identity element(s) then it is a "Monoid".

Ex:

$(N,*)$ = semigroup (✓) $a*e = a$ Now $\boxed{e=1}$ ∴ $(N,*)$ is Monoid

and '1' is the identity element of N w.r.t. $*$

$(N,+)$ = AS (✓) SG (✓) $a+e = a$ ⇒ $\boxed{e=0 \notin N}$ ∴ This is not Monoid.

$(Z,+)$ = AS(✓) SG(✓) $a+e=a$ ⇒ $\boxed{e=0} \in Z$ ∴ Monoid

$(P(A),\cup)$ = AS(✓) SG (✓) $X \cup e = X$ $\boxed{e=\phi \in P(A)}$ ∴ Monoid.

Algebraic Structures (Closure property)
↓
Semi Groups (Associative property)
↓
Monoid (Identity elements)

4. GROUP

A monoid $(S,*$ ...
each element ...
$(b*a)=e$ then ...

Ex

$(Z,+)$ = Monoid ...

$(Q,\cdot)$ • = (Multip...

$(Q^*,\cdot)$ ⇒ Q^* = (set o...
no's ...
• = Multi...

$(P(A),\cup)$ = $X \cup \{\phi\}$ ...

5. ABELIAN GRO...

→ In a Group (...
i> The Identity ...
ii> The Inverse o...
iii> The Inverse o...
iv) Cancellation la...

v) $(a*b)^{-1} = b^{-1}*a^{-1}$

6. EXAMPLE 1

Abelian Group (Com...

A group $(G,*)$ is ...

Ex:

$(Z,+)$ = Group(✓) a...
$(R^*,\cdot)$ = Group(✓) ...
$(M,*)$ = M = (set of all ...

4. GROUP

A monoid $(S,*)$ with identity element 'e' is called a group if to each element $a \in S$, there exists an element $b \in S$, such that $(a*b) = (b*a) = e$ then 'b' is called Inverse of an element a, denoted by a^{-1}.

Ex

$(Z,+)$ = Monoid(✓) $= a+b = b+a = 0$ (Identity element). $= \boxed{a=-b}$ and $(-b) \in Z$ ∴ Group | For every ele there is Inverse

$(Q,\cdot)$ $\cdot$ = (Multiplication) $= a*b = 1$ ∴ This is a Not Group

$\Rightarrow \boxed{b = (1/a)} \notin Q$ if $(a=0)$

$(Q^*,\cdot) \Rightarrow Q^*$ = (set of all Rational no's without '0')

$\cdot$ = Multiplication

$a*b=1$

$\Rightarrow \boxed{a = 1/b}\ \boxed{b = 1/a} \in Q^*$

$\Rightarrow$ This is Group because the set Q^* doesnot has '0'.

$(p(A),\cup) = X \cup \{\phi\} = \{\phi\} \Rightarrow$ identity element $\Big\} =$ Group.

↳ $\boxed{\phi \in p(A)}$

5. ABELIAN GROUP.

→ In a Group $(G,*)$ the following properties must hold good

i> The Identity element of 'G' is unique

ii> The Inverse of any element in 'G' is unique

iii> The Inverse of identity element 'e' is 'e' itself

iv) Cancellation laws $(a*b) = (a*c) \Rightarrow b=c$

$(a*c) = (b*c) \Rightarrow a=b$

v) $(a*b)^{-1} = b^{-1}*a^{-1}\ \forall\ a,b \in G$

6. EXAMPLE 1

Abelian Group (commutative Group)

A group $(G,*)$ is said to be Abelian if $(a*b) = (b*a)\ \forall\ a,b \in G$.

Ex:

$(Z,+)$ = Group(✓) $a+b = b+a \in Z \Rightarrow$ Abelian Group

$(R^*,\cdot)$ = Group(✓) $a*b = b*a \in R^* \Rightarrow$ Abelian Group.

$(M,*)$ = M = (set of all non Singular matrices) $\Rightarrow$ ($*$ = matrix multiplication) = NOT Abelian Group

7. EXAMPLE 2

Which of the following is/are True?

1) In a group $(G, *)$ with an identity element 'e' if $a*a = a$ then $a = e$

2) In a Group $(G, *)$ if $x^{-1} = x \ \forall x \in G$ then 'G' is Abelian Group.

3) " " " " " $(a*b)^2 = a^2 * b^2 \ \forall a, b \in G$ then 'G' is Abelian Group.

⇒ Use the formulas in 5th video. Now, $a*a = a$

1) $a*\cancel{a} = \cancel{a}*e$ (I can write 'a' as $a*e$ because 'e' is identity element)

$\boxed{a = e}$ TRUE.

2) $x^{-1} = x$

$x^{-1} = x * e$

$(a*b)^{-1} = b^{-1} * a^{-1}$

$\boxed{(a*b) = (b*a)}$ ⇒ Given of $x^{-1} = x$ so $(a*b)^{-1} = (a*b)$

$b^{-1} = b$

$a^{-1} = a$

3) $a^2 = a * a$ write the operation specified.

⇒ $(a*b)^2 = a^2 * b^2$

$= \cancel{(a*b)}*\cancel{(a*b)}$ $\quad \underbrace{(a*b)*(a*b)} = a*a * b*b.$

$= (a*a)*(b*b) = (a*a)*(b*b) \quad$ = LHS = RHS.

$\cancel{a}*(b*a)*\cancel{b} = \cancel{a}*a*b*\cancel{b}$

$(b*a) = (a*b)$

8. EXAMPLE - 3

If $A = \{1, 3, 5, 7, 9 --- \infty\}$ and $B = \{2, 4, 6, 8 --- \infty\}$ which of the following is semigroup?

a) $(A, +)$ = Algebraic structure (✗)

~~b)~~ $(A, \cdot)$ = AS(✓) SG(✓) Monoid (✓) $(e = 1)$ $\quad a * \{x\} = 1$ $\quad x = 1/a \notin A$ ∴ Not Abelian.

~~c)~~ $(B, +)$ = AS(✓) SG(✓)

~~d)~~ $(B, \cdot)$ = AS(✓) SG(✓)

9. EXAMPLE-4

Let $A = \{1,2,3,4 --- \infty\}$ and a Binary operation $*$ is defined by $a*b = a^b$ $\forall a,b \in A$ which of the following is true?

a) $(A,*)$ is SemiGroup but not monoid AS(✓) SG(×)

b) $(A,*)$ is monoid but not group

c) $(A,*)$ is Group

d) $(A,*)$ is not semi Group.

$a*(b*c) = (a*b)*c$

1 2 3 | 1 2 3

$= 1*(2^3)$ | $(1^2)*3$

$= 8$ | $= (1*3) = 1^3$

$= 1$

$(8 \neq 1)$

so not SG

10. EXAMPLE-5

Let $A = \{x / 0 < x \leq 1$ and x is a Real number$\}$ then 'A'. w.r.t to multiplication is

$x \in (0,1]$

a) A semigroup but not monoid

b) A monoid but not Group

c) A group

d) Not a semigroup

$(A,*) \Rightarrow$ AS(✓). semiGroup. (✓)

$(a*b)*c = a*(b*c)$

, 0.1 0.1 = , 0.1 0.1

$= (0.1)^2 = (0.1)^2$

Monoid (✓)

$\boxed{e=1}$ Identity element

Group.

$a*\{x\} = e$

$\Rightarrow \boxed{x = 1/a \notin A}$

$\frac{1}{0.1} = 10 \notin A$ Group(×).

11. EXAMPLE-6

Let 'A' is set of all integers and Binary operation $*$ is defined by $(a*b) = \min(a,b)$, then $(A,*)$ is Semigroup

$(z, \cdot) \Rightarrow$ Alegebraic Structure(✓)

semiGroup (✓)

$(a*b)*c = a*(b*c)$

1 2 3 | 1 2 3.

$= 1*3$ | $1*(2)$

$= 1$ | 1

Monoid

$\boxed{a*\{e\} = a}$

$\boxed{e=1}$ identity element

$a*e = a$.

$a*e = \min(a,e)$

101 100 $= 100 \neq 101$

Group.

$a*b = a \Rightarrow a*? = e$

$\Rightarrow a*? = 1$

$? = 1/a \notin Z$

SemiGroup. Monoid

$\therefore$ $\boxed{\text{It is } \cancel{\text{Monoid}} \text{ but not } \cancel{\text{Group}}}$

16. FINITE GROUPS.

A Group with finite no. of elements is called finite Groups.

$O(G) \rightarrow$ order of finite Group. (No. of ele in the Group)

Ex:

1) $(\{0\}, +) = (0+0) = 0$, Identity element = '0'. (Monoid) Group (✓)

$\Rightarrow$ $\boxed{\text{whenever a group is having only one element then that ele will be the Identity element. (V.V. Imp)}}$

2) $(\{1\}, *) = 1*1 = 1 \in$ set (AS) ✓.

$(1*1)*1 = 1*(1*1)$ (SG) ✓

$a*e = a = 1*e = 1 \Rightarrow \boxed{e=1} \in$ set (monoid) ✓

$a*\{x\} = e \Rightarrow x = 1/a \quad x = 1/1 \Rightarrow \boxed{x = 1 = e}$ $\therefore$ (Group)

3) $(\{1, -1\}, *)$ = Group (✓)

= composition table

*	1	-1
1	1	-1
-1	-1	1

$\in$ (set) $\therefore$ (AS) ✓

= (SG) ✓

= Monoid ✓ (e = 1 Identity element)

= Group.

4) $\{(1, \omega, \omega^2), *\}$ =

*	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

AS (✓)

$\rightarrow$ Identify ele = 1 (mon) ✓

(SG) ✓

$\omega^4 = \omega^3 \times \omega$

$= 1 \times \omega$

$\boxed{\omega^4 = \omega}$

For all the ele we are able to find inverses $\therefore$ Group

*	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

*	1
1	1

$\rightarrow$ 1 is the Inverse of 1

*	ω^2
ω	1

ω^2 is the Inverse of ω

*	ω
ω^2	1

'ω' is the Inverse of ω^2

EXAMPLES ON FINITE GROUPS

1) If $G = \{1,3,5,7\}$ is a group w.r.t $\times_8$ (Multiplication modulo 8), which of the following is not true?

a) The inv of 1 is 1

b) The Inv of 3 is 3

(c) The inverse of 5 is 7

(d) The inverse of 7 is 7.

Now, $1 \times 1 = 1$ (1 when divided by 8 gives '1' as Remainder)

$3 \times 3 = 9$ ($9 \div 8 = 1$ (belong to G) and 1 is identity element)

$5 \times 5 = 25$ ($25 \div 8 = 1$ (identity element)

$7 \times 7 = 49$ ($49 \div 8 = 1$ (identity element).

$\therefore$ Each element is Inverse of itself

(a)

	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

1 is the Inverse of 1
3 " " " " 3
5 " " " " 5
7 " " " " 7

How did i write this is $(7 \times 5) \div 8 = 35 \div 8 = 3$.

2) Which of the following is a Group?

a) $\{1,2,3,4\}$ w.r.t $\times_6$

b) $\{0,1,2,3,4,5\}$ wrt $\times_6$

c) $\{1,2,3,4,5,6\}$ wrt $\times_7$

d) $\{1,2,3,4,5,6\}$ wrt $+_7$

we know that

$\oplus_m$ contains $0,1,2 \ldots (m-1)$ elements

$\otimes_m$ contains 'm' elements.

a) $\{1,2,3,4,5\}$ w.r.t $\times_6$

$= (2 \times 3) \div (6)$

$\Rightarrow 0 \notin$ set

$\Rightarrow$ (AS) ✗.

(b) $\{0,1,2,3,4,5\}$ wrt $\times_6$

$\Rightarrow$ NO Inverse to '0' and we cannot get Identity element so doesn't form Group.

(c) $\underbrace{\{1,2,3,4,5,6\}}_{S_7}$ wrt $\times_7$

S_7 (set of all no's less than 7 and are relatively prime to 7).

$\therefore$ Group

(d) $\{1,2,3,4,5,6\}$ wrt $\oplus_7$

(0, 6) must be present

'0' is missing.

Monoid property fails

18. EXAMPLES ON FINITE GROUPS

17

1) $(\{0,1,2 \ldots (m-1)\}, \oplus_m)$ Addition modulo 'm' (This will always be Group)

$$a \oplus_m b \begin{cases} \text{If } (a+b) < m \Rightarrow (a+b) \text{ is the result} \\ \text{If } (a+b) > m \Rightarrow (a+b)\,\%\,m \text{ is the result} \end{cases}$$

$(\{0,1,2\}, \oplus_3) \Rightarrow$ This is a group

$\oplus_3$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$(0,1,2) \in$ set $\therefore$ AS✓ (SG)✓ (m)✓ (G)✓

⇒ For getting the identity element check the row containing {0,1,2} (in the same order) then that element will be Identity element here '0' row has {0,1,2} ∴ '0' is the identity element

⇒. Here In this case check for 0's

	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$a \times a^{-1} = e$

$a \times a^{-1} = 0$

$\boxed{a^{-1} = 0}$

0 ⇒ 2 is the Inverse of 1.

↳ 1 is the Inverse of 2.

2) $(S_m, \otimes_m) \Rightarrow$ $\boxed{a \otimes_m b = (a \times b)\,\%\,m.}$

↓

Set of all no's that are less than 'm' and relatively prime to 'm'.

$S_{10} = \{1, 3, 7, 9\}$ ∴ $S_{10} = \{1,3,7,9\}$.

Two no's are said to be Relatively prime if GCD (a,b) = 1

Now, $\{S_{10}, \otimes_{10}\}$ = Group.

$= \{S_{10}, \otimes_{10}\} = \{\{1,3,7,9\}, \otimes_{10}\}$ = Group.

19. ORDER

Order of an element of a group

Let $(G, *)$ be a Group and $a \in G$, then order of element 'a' is the smallest positive integer 'n' such that a^n is identity element.

Ex:

1) $(\{1, -1\}, *)$ Identity element $= 1$

Now, $(-1)^2 = 1 =$ identity element $\therefore$ order of $(-1) = 2$

$(-1)^4 = 1$ '4 cannot be order we should always take least no).

$\Rightarrow$ order of Identity element is always one.

2) $(\{1, \omega, \omega^2\}, *)$ Identity element $= 1$

$1^1 = 1 \Rightarrow$ order of $1 = 1$

$\omega^3 = 1 \Rightarrow$ order of $\omega = 3$.

~~$\omega^2 \times \omega \neq 1 \Rightarrow$ order of $\omega^2 = 1$~~ Now, $(\omega^2)^3 = \omega^6 = (\omega^3)^2 = 1$

$\therefore \boxed{O(\omega^2) = 3 \quad O(\omega) = 3 \quad O(1) = 1}$

$\Rightarrow$ Order of any element divides the order of group (Finite Group)

3) $(\{0, 1, 2\}, \oplus_3)$ Identity element $= 0$.

Now, $0^1 = 0$. $\therefore O(0) = 1$

$1 \oplus_3 1 \oplus_3 1 = 0 \Rightarrow \boxed{O(1) = 3} \Rightarrow \boxed{1^3 = 0}$

$2 \oplus_3 2 \oplus_3 2 = 0 \Rightarrow \boxed{O(2) = 3} \Rightarrow \boxed{2^3 = 0}$

In a Group $O(a)$ and $O(a^{-1})$ are always same/Equal.

20. EXAMPLES ON ORDER

State True / False

S1) In a group $(Z, +)$ the order of any element except '0' doesnot exist

S2) In the group $(Q^*, \cdot)$ where Q^* is set of all Non-zero rational numbers i.e $Q^* = Q - \{0\}$, the order of any element except 1 doesnot exist

S1) TRUE, becuase order of '0' is 1 (because '0' is Identity element) and for other ele order doesnot exist

S2) 1 is the Identity element so $O(1) = 1$

Now, $(-1)^2 = 1$ $\therefore \boxed{O(-1) = 2}$ $\therefore$ S2 is FALSE.

21. SUBGROUPS.

SUBGROUPS.

Let $(G, *)$ be a group. A subset 'H' of 'G' is called a SubGroup of 'G' if $(H, *)$ is a group.

Ex:

let $(G, *)$ be a group with Identity element 'e', then $\{e\}$ and 'G' are the trivial SubGroups of 'G'. Any subgroup which is not a trivial subgroup is called proper SubGroup.

$G = (\{1, -1, i, -i\}, *)$ then $H = (\{1, -1\} *)$ is a proper subgroup.

$\Rightarrow$ Every Group is going to have atleast two subgroups 1> Itself 2> set containing Identity elements. (These are called Trivial subGroups)

The SubGroups other than Trivial SubGroups are called propersubGrp.

22. THEOREMS ON SUB GROUPS.

Th 1: Let 'H' be non empty subset of a group $(G,*)$ 'H' is a sub group of 'G' iff $a*b^{-1} \in H \ \forall a,b \in H$

Th 2: Let 'H' be non empty finite subset of a group $(G,*)$ 'H' is a subgroup of 'G' iff $(a*b) \in H \ \forall a,b \in H$

Th 3: Lagranges Theorem :-

If 'H' is a subgroup of finite group $(G,*)$ then $O(H)$ is the divisor of $O(G)$. The converse of the above theorem need not be True.

$\left.\begin{array}{l} O(G) = m \\ O(H) = n \end{array}\right\} \Rightarrow$ m is divisable with/by 'n'. (This is what this theorem says). Ex: $\left.\begin{array}{l} O(G) = 10 \\ O(H) = 3 \end{array}\right\}$ 'H' cannot be subgroup of 'G' becuase '3' does not divide 10.

23. EXAMPLES ON SUBGROUPS - 1

Let $G = (\{0,1,2,3,4,5\}, \oplus_6)$ is a group. which of the following is/are subgroups of G?

a) $H_1 = \{1,3\} \Rightarrow$ Now, $\{\{1,3\} \oplus_6\}$ $(1+3) \oplus_6 = 4 \bmod 6 = 4 \notin H_1$

b) $H_2 = \{1,5\} \Rightarrow$ Now, $(\{1,5\}, \oplus_6) = (1+5) \bmod 6 = 0 \notin H_2$

c) $H_3 = \{1,3\} \Rightarrow$ Now, $(\{1,3\}, \oplus_6) = (4) \bmod 6 = 4 \notin H_3$.

d) $H_4 = \{0,2,4\} \Rightarrow$ Now, $(\{0,2,4\}, \oplus_6) =$ $\begin{array}{l} 2 \bmod 6 = 2 \\ 0 \bmod 6 = 0 \\ 4 \bmod 6 = 4 \end{array}$ $\left.\begin{array}{l} (0+2) \bmod 6 = 2 \\ (0+4) \bmod 6 = 4 \\ (2+4) \bmod 6 = 0 \end{array}\right\} \in H_4$

$\Rightarrow$ subGroup

e) $H_5 = \{0,2,3,5\}$.

Now $(\{0,2,3,5\}, \oplus_6)$

$= 0 \bmod 6 = 0$

$= 2 \bmod 6 = 3 \bmod 6$

$= 5 \bmod 6 = 2,3,5$ respectively

$\Rightarrow 0,2,3,5 \in H_5$

$(0,2) \bmod 6 = 2$

$(0,3) \bmod 6 = 3$

$(0+5) \bmod 6 = 5$

$(2+3) \bmod 6 = 5$

$(2+5) \bmod 6 = 1 \notin H_5$

$\therefore$ This wont be subGroup.

(OR) "PROVE BY CONSTRUCTING COMPOSITION TABLE"

24. EXAMPLES ON SUB GROUPS - - 2

$G = (\{1,2,3,4,5,6\}, \circledast_7)$ which of the following are subgroups of 'G'.

a) $H_1 = \{1,6\}$ c) $H_3 = \{1,3,5\}$

b) $H_2 = \{1,2,4\}$ d) $H_4 = \{1,2,3,5\}$.

H_1:

	1	6
1	1	6
6	6	1

H_1 is sub group

H_2:

	1	2	4
1	1	2	4
2	2	4	1
4	4	1	2

$\rightarrow \in H_2$

H_2 is subGroup.

H_3:

	1	3	5
1	1	3	5
3	3	2	1
5	5	1	4

$2 \notin H_3$.

$\therefore H_3$ is not sub Group.

H_4:

	1	2	3	5
1	1	2	3	5
2	2	4	6	3
3	3	6	2	1
5	5	3	1	4

$\rightarrow 6 \notin H_4$ $\therefore H_4$ is not subGroup.

25. EXAMPLES ON SUB GROUPS - - 3

Let $(G,*)$ be a group of order p where 'p' is prime no, then the no. of proper subgroups of 'G' is ___ ?

Sol: Given the order of 'G' has prime number = p (1 and 'p' are only factors)

$\therefore$ The subgroups of G has the order '1 and 'p'.

$\Rightarrow$ so 2 subgroups are possible that are the total set $(G,*)$ and the Identity element set say $(e,*)$

$\longrightarrow$ But these are Trivial subgroups.

$\therefore$ The total no. of subgroups = Total - Trival subGroups

$= 2 - (2)$

$\boxed{\text{SubGrps} = 0}$

26. EXAM

Which of th

a) The union

b) The Intersec

c) The union

d) Every subg

a) $(S_8, \otimes_8)$

b) TRUE. (same

c) FALSE (Refe

d)

26. EXAMPLES ON SUBGRAPHS -- 4

Which of the following are not true? (FALSE)

a) The union of any two subgroups of 'G' is also a subgroup of 'G'.

b) The intersection " " " a " " " " " " "

c) The union of 2 subgroups H_1 and H_2 of group $(G,*)$ is also a subgroup of 'G'.

d) Every subgroup of an Abelian group is also Abelian group.

a) $(S_8, \otimes_8) = (\{1,3,5,7\}, \otimes_8)$

= subgroups $H_1 = \{1,3\}$ $H_2 = \{1,5\}$

	1	3
1	1	3
3	3	1

	1	5
1	1	5
5	5	1

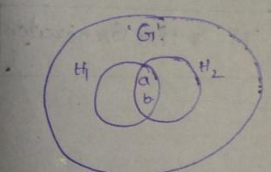
= union of H_1 and H_2 = $\{1,3\} \cup \{1,5\} = \{1,3,5\}$

	1	3	5
1	1	3	5
3	3	1	7
5	5	7	4

$\notin (H_1 \cup H_2)$

$\therefore$ $\boxed{\text{a is false}}$

b) TRUE. (same above example) $\{1,3\} \cap \{1,5\} = \{1\}$

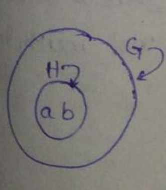


	1
1	1

$\rightarrow \in H_1 \cap H_2$ $\therefore$ SubGroup.

c) FALSE (Refer (a))

d)



Given 'G' is Abelian Group and hence 'H' which is subset of Abelian Group is again Abelian Group.

$\boxed{\text{Every subset of Abelian Group is Abelian.}}$

27. CYCLIC GROUPS

A group $(G,*)$ is called a cyclic Group if there exists an element $a \in G$ such that every element of 'G' canbe written as a^n for some integer n. Then 'a' is called generating element/generator.

1) $G = (\{1,-1\},*)$ Now $(-1)^1 = (1)^1 = -1$, $(-1)^2 = 1$ — The elements can be generated using (-1) as $(-1)^1$ and $(-1)^2 = 1$.

$\therefore$ $\boxed{(-1) \text{ is the GENERATOR}}$

2) $G = (\{1,\omega,\omega^2\},*)$ Now, $\omega^1 = \omega$, $\omega^2 = \omega^2$, $\omega^3 = 1$ — all the elements can be generated using "ω"

$\therefore$ $\boxed{\text{Generator} = \omega}$

3) $G = (\{1,-1,i,-i\},*)$ Now, $i = i^1 = i$

$i^2 = -1$

$(i^2)^2 = i^4 = 1$ $\therefore$ $\boxed{\text{'i' is the Generator}}$

$i^3 = -i$

4) $G = (\{0,1,2,3\},\oplus_4)$ Now,

$1^1 = 1$

$1^2 = 1+1 = 2$

$1^3 = 1+1+1 = 3$

$1^4 = 0 \quad (4 \bmod 4 = 0)$

Operation defined here is addition not multiplication

Now, $3^1 = 3$

$3^2 = 6 \bmod 4 = 2$

$3^3 = 9 \bmod 4 = 1$

$3^4 = 12 \bmod 4 = 0$

'3' is also a Generator

EXAMPLES ON CYCLIC GROUPS

→ If $(G, *)$ is a cyclic group with generator 'a' then

i) a^{-1} is also a generator

ii) The order of the Generator $= O(G) =$ Order of the Group.

Ex: $(\{0, 1, 2, 3\}, \oplus_4)$

$1^1 = 1 \quad 1^2 = 2, \quad 1^3 = 3 \quad 1^4 = 4 \bmod 4 = 0$. $\therefore$ 1 is the generator

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	4	1	2

Now, the Inverse of $a * a^{-1} = e$ (Identity ele)

Here 0, is the Identity element $\Rightarrow \cancel{1 * a^{-1} = 0}$

$\Rightarrow \cancel{a^{-1} = 0}$

$\therefore$ check in 1 row where '0' is present

$\therefore$ under '3' numbered column.

$\therefore$ $\boxed{\text{Inverse of 1 is '3'}}$

$\therefore 3^1 = 3$

$\therefore 3^2 = 2$

$3^3 = 1$

$3^4 = 0$

$\therefore$ 3 is also a Gerator.

$\boxed{\text{1st point is satisfied}}$

$\therefore O(1) = 4$ (because $1^4 = 0$ (identity element)

$O(3) = 4$ $(3^4 = 0$ (Identity element).

Ex: $(S_5, \cdot 5) = (\{1, 2, 3, 4\}, \otimes_5)$

$2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 3, \quad 2^4 = 1$

$3^1 = 3, \quad 3^2 = 4, \quad 3^3 = 2, \quad 3^4 = 1$

2 and 3 are generators

	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Now, the identity element is '1' Now

'3' is the Inverse of '2'.

$\left.\begin{array}{l} O(3) = 4 \\ O(2) = 4 \end{array}\right\} = O(G) = O(S)$.

29. THEOREM ON CYCLIC GROUPS

Let $(G,*)$ be a cyclic group of order 'n' with generator 'a' then

1) The no. of generators in $G = \phi(n)$ → Eulers function of 'n'.

2) a^m is also generator of G if $GCD(m,n) = 1$

$\boxed{\phi(n) = \text{The no. of numbers that are less than 'n' and relatively prime to n.}}$

Ex

Let $(G,*)$ be a cyclic group of order 8 with generator 'a'

(i) No. of generators in $G = ?$ $\underline{4}$

(ii) which of the following is not a generator of G ?

(a) a^2 (b) a^3 (c) a^5 (d) a^7

Now, Here $n=8 \Rightarrow$ The set $\phi(n) = \{1,3,5,7\}$

$\therefore$ No. of generators in $\boxed{G = 4}$

Now, $n = 8$. Now, $\because a^m$ is also generator if $GCD(m,n) = 1$

$\Rightarrow a^2 = GCD(2,8) \neq 1$ — NOT Generator

$\Rightarrow a^3 = GCD(3,8) = 1$ — Generator

$\Rightarrow a^5 = GCD(5,8) = 1$ — Generator

$\Rightarrow a^7 = GCD(7,8) = 1$ — Generator.

The problem with this method is as the value of 'n' increases then it is difficult to find the prime no/s less than 'n'. So the product rule has been defined.

If $n = p \times q$ (If 'n' can be written as product of two distinct prime numbers 'p' and 'q' then)

$\boxed{\phi(n) = \phi(p)\,\phi(q)}$

The Advantage with this method is if 'p' is a prime number then $\boxed{\phi(p) = (p-1)}$ $[\phi(7) = 6, \phi(11) = 10]$

Now, $\phi(77) = \phi(7) \times \phi(11)$

$= 6 \times 10 = 60$ //.

Now, In the above procedure Both 'p' and 'q' should be distinct if they are same prime numbers then the formula is

$$\boxed{\phi(p^n) = p^n - p^{n-1}}$$

'p' should be prime

$\phi(25) = \phi(5^2) = 5^2 - 5$

$\boxed{\phi(25) = 20}$

Now, $\phi(84) = 2 \times 2 \times 3 \times 7$

$\phi(84) = \phi(2^2 \times 3 \times 7)$

$= \phi(2^2) \times \phi(3) \times \phi(7)$

$= (2^2 - 2) \times (2) \times (6)$

$\left[\begin{array}{l} \phi(3) = (3-1) = 2 \\ \phi(7) = (7-1) = 6 \end{array}\right.$ $\phi(2^2) = 2^2 - 2^1 = 2$

$= 2 \times 2 \times 6$

$\boxed{\phi(84) = 24}$

EXAMPLES ON CYCLIC GROUPS

1) $G_1 = (\{1,2,3,4,5,6\}, \otimes_7)$ Find all the generators of G_1, G_2, G_3.

2) $G_2 = (\{0,1,2,3,4\}, \oplus_5)$

3) G_3 $(\{1,3,5,7\}, \otimes_8)$

Now, $G_1 = (\{1,2,3,4,5,6\}, \otimes_7)$

No. of generators $= \phi(6) = \{1,5\} = 2$ generators $\Rightarrow a^1, a^5$ are generators

Now, 1 cannot be generator because it is identity element.

2 is not generator

$2^1 = 2$ $\quad 2^4 = 2$

$2^2 = 4$ $\quad 2^5 = 4$

$2^3 = 1$ $\quad 2^6 = 1$

} 4, 5, 6 are not generated

3 is generator.

$3^1 = 3$ $\quad 3^5 = 3^2 \times 3^3$

$3^2 = 2$ $\quad = 2 \times 6$

$3^3 = 6$ $\quad = 5$ (12 mod 7)

$3^4 = 3^2 \cdot 3^2 = 4$ $\quad 3^6 = 3^3 \times 3^3$

$= 6 \times 6$

$= 1$ (36 mod 7)

The 2 generators are 3^1 and 3^5

$= 3$ and 5

Now,

(2) $G_2 = (\{0,1,2,3,4\}, \oplus_5)$ $o(G) = (5)$

No. of generators $= \phi(5) = \{4\} = \{0,1,2,3,4\} \Rightarrow a^0, a^1, a^2, a^3, a^4$ are generators

Now, 0 is the identity element $\Rightarrow$ it cannot be generator

$1^1 = 1$ $1^2 = 2$ $1^3 = 3$ $1^4 = 4$ $1^5 = 0$. 1 is generator.

$= 1^1 \quad 1^2 \quad 1^3 \quad 1^4$ are generators.

$=$ | 1 | 2 | 3 | 4 | are generators

(3) $G_3 = (\{1,3,5,7\}, \otimes_8)$

Now, $\phi(4) = \{1,3\}$. $\therefore$ a^1, a^3 are the 2 generators.

Now 1 is Identity element

$3^1 = 3$ $3^2 = 1$ $3^3 = 3$ $3^4 = 1$ $\therefore$ 3 is not generator

$5^1 = 5$ $5^2 = 1$ $5^3 = 5$ $5^4 =$ not generator

$7^1 = 7$ $7^2 = 1$ not generator.

$\therefore$ There are no generators for this group $\Rightarrow$ The group is not cyclic group.

31. SOME POINTS ON CYCLIC GROUPS.

For cyclic groups, the following properties hold good. g = generator (assume)

1) Every cyclic group is an Abelian Group $[a*b = g^n \times g^m = g^{m+n} = g^m \times g^n = b*a]$

2) Every Group of prime order is cyclic and so every group of prime order is Abelian Group.

3) Every subgroup of a cyclic group is also cyclic, but the generator of the subgroup need not be same as that of cyclic group.

   Ex: $G = \{1, -1, i, -i\}$; $H = \{1, -1\}$ so, H is subgroup of G'

   Generators of G are $i, -i$, Generator of $H = -1$

4) Let $(G, *)$ be a group of even order, then there exists atleast one element $a \in G$ $(a \neq e)$ such that $a^{-1} = a$.

5. FUNCTIONS

INTRODUCTION TO FUNCTIONS

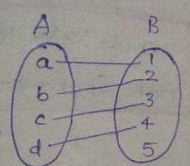
A Relation 'f' from 'set A' to a 'set B' is called a function if to each element $a \in A$, we can assign an unique element of 'B'. It is denoted as $f: A \to B$. A is domain and 'B' is co-Domain.

Range: Range of function is $\{y / y \in B \text{ and } (x,y) \in f\}$ so range of $f \subseteq B$.

A function $f: A \to A$ is called a function on 'A'.

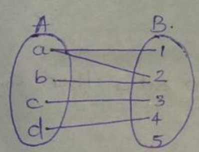
$f: (A \to B)$

Range $= \{1, 2, 3, 4\}$



$\{(a,1)(b,2)(c,3)(d,4)\}$

Image of a = 1
b = 2
c = 3
d = 4



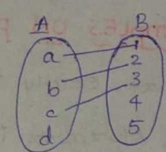
Not a function

$\{(a,1)(a,2)\}$

Same ele is mapped to two elements.

R

$f: (A \to B)$

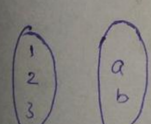


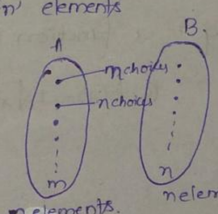
Not function

'd' is not mapped.

COUNTING THE FUNCTIONS

$f: A \to B$ Assume 'A' has 'm' elements
'B' has 'n' elements





1 has 2 choices (1,a) (1,b)

2 has 2 choices (2,a)(2,b)

3 has 2 choices (3,a)(3,b)

$n \times n \times n \times \cdots$ (m) times

$= n^m$

$\therefore$ $\boxed{\text{The no. of functions from } A \to B = n^m}$

$= (\text{No. of ele in B})^{\text{No. of ele in A}}$

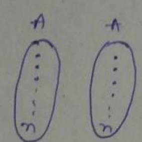
Now,

The total no. of Relations from $A \to B$ which are not functions is

The no. of Relations that are not functions

$$= \boxed{2^{mn} - (n)^m}$$

Now, if we are defining the function on the same set $f: A \to A$ then, The total no. of Relations that are not functions are



No. of Relations $= 2^{n^2}$

No. of functions $= n^n$

$\therefore$ No. of Relations that are not functions $= \boxed{2^{n^2} - n^n}$

3. EXAMPLES ON FUNCTIONS

If there are exactly 81 functions from set A to set B then which of the following statements is not True?

a) $|A| = 4$ $|B| = 3$

b) $|A| = 2$ $|B| = 9$

c) $|A| = 1$ $|B| = 81$

d) $|A| = 9$ $|B| = 9$.

No. of functions from $A \to B = (\text{No. of ele in } B)^{\text{No. of ele in } A}$.

option 1 $\Rightarrow 3^4 = 81$ ✓ function

2 $\Rightarrow 9^2 = 81$ ✓

3 $\Rightarrow 81^1 = 81$ ✓

4 $\Rightarrow 9^9 \neq 81$. $\therefore$ option '4' is false.

Which of the following is a function if domain is set of all real no's.

a) $f(x) = 1/x$ (b) $f(x) = \sqrt{x}$ (c) $h(x) = \pm\sqrt{x^2+1}$ d) $\phi(x) = |x|$

Domain = All Real no's.

a) $f(x) = 1/x$ If $x = 0$?? then the image will not be present for '0' so this is not a function.

b) $f(x) = \sqrt{x}$: for $-ve$ no's square root is not possible (Not function)

c) Not function because for every element there are two values $+x$ and $-x$.

d) function.

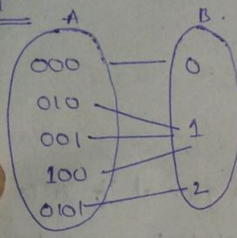
→ Consider the following relations from set of all bit strings to set of all integers.

$f_1(S)$ = The no. of 1's in the bit string 'S'.

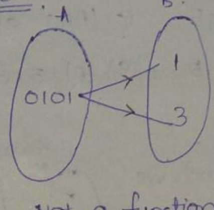
$f_2(S)$ = The position of a 0-bit in a bit string 'S'.

which of the above relations are functions.

$f_1(S)$



$f_2(S)$



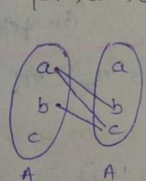
['0' is present at position '1' and '3' so it will be mapped to 2 elements in B.

Not, a function

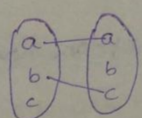
4. EXAMPLES ON FUNCTIONS

Which of the following relations on set A is a function? $A = \{a, b, c\}$

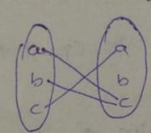
a) $R_1 = \{(a,b)(b,c)(a,c)\}$ (b) $R_2 = \{(a,a)(b,c)\}$ c) $R_3 = \{(a,c)(b,c)(c,a)\}$



NOT function



Not function



Function.

b) State True / False ?

S1) There exists equalence Relation which is function. TRUE $\{(1,1)(2,2)(3,3)\}$.

S2) The functions $f(x) = x$, $g(x) = \sqrt{x^2}$ are identical FALSE

S3) $f(x) = \log_e x^2$ and $g(x) = 2\log_e x$ are identical. FALSE

S4) The domain of $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is $(-\alpha, 0)$ $\Rightarrow |x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$ TRUE.

if $x > 0 \Rightarrow f(x) = 1/0$ ∴ $f(x)$ is not defined for +ve nos

∴ Domain $= (-\alpha, 0)$

S2

$f(x) = x$	$g(x) = \sqrt{x^2}$
$f(1) = 1$	$g(1) = 1$
$f(-1) = -1$	$g(-1) = 1$

$(-1 \neq 1)$ ∴ NOT IDENTICAL

S3

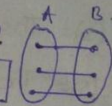
$f(x) = \log_e x^2 \Rightarrow f(-1) = \log_e (-1)^2 = \log_e 1 = 0$

$g(x) = 2\log_e x = 2\log_e(-1)$ = does not exist.

$g(-1) = \text{?}$

5. ONE-ONE FUNCTIONS (INJECTION)

A function 'f' from a set 'A' to set 'B' is said tobe one-to-one if no two elements in 'A' are mapped to same element in 'B'. $\boxed{|B| \geq |A|}$

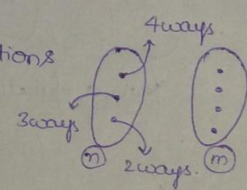


→ If there are exactly 120 one-to one functions possible from A to B then which of the following is not true?

a) $|A| = 5$ and $|B| = 5$ (c) $|A| = 3$ and $|B| = 6$

b) $|A| = 4$ and $|B| = 5$ (d) $|A| = 5$ and $|B| = 4$

No. of one-to one functions



∴ In general if 'A' is having 'n' elements and B is having 'm' elements then

the no. of one-one function from $A \to B = (m)(m-1)(m-2)(m-3)\ldots(m-n+1)$

$$= {}^{m}P_{n} = \boxed{(\text{No of ele in B})_{P_{(\text{no. of ele in A})}}}$$

if m, n are equal then the no. of one-one functions $= {}^{n}P_{n} = n!$

Now, In the Question (No. of one-one functions $= {}^{m}P_{n} = 120$)

→ op1 : $|A| = 5$ $|B| = 5$ $= {}^{5}P_{5} = 5! = 120$. TRUE

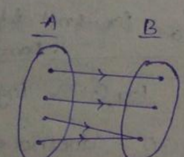
⇒ op2 : $|A| = 4$ $|B| = 5$ $= {}^{5}P_{4} = 5 \times 4 \times 3 \times 2 = 120$. TRUE

⇒ op3 : $|A| = 3$ and $|B| = 6$ ⇒ ${}^{6}P_{3} = 6 \times 5 \times 4 = 120$.

⇒ op4 : $|A| = 5$ $|B| = 4$ (${}^{4}P_{5}$) — Not possible

6. ONTO FUNCTIONS

A function $f: A \to B$ is said tobe onto if each element of 'B' is mapped by atleast one element of 'A'. i.e Range of $f = B$.



onto function

The condition for a function tobe onto is $\boxed{|B| \leq |A|}$ ⇒ If this doesnt satisfy then func is not onto.

→ we cannot say a func is onto if it satisfy the above condition.

If $|A| = |B|$ then the no. of onto functions $= n!$

7. EXAMPLES ON ONTO FUNCTIONS - 1

If $|A| = m$ and $|B| = n$, $(m>n)$ then the no. of onto fundions possible from A to B is.

$$\boxed{n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - {}^nC_3 (n-3)^m + \dots + (-1)^n \, {}^nC_{n-1} (1)^m}$$

Ex: If $|A| = 6$, $|B| = 3$ then the no. of onto functions from A to B is —?

$|A| = 6 \Rightarrow m = 6$

$|B| = 3 \quad n = 3.$

$\Rightarrow 3^6 - {}^3C_1 (2)^6 + {}^3C_2 (1)^6 - {}^3C_3 (0)^6 = 729 - 3\times 64 + 3\times 1 = \underline{\underline{540}}$

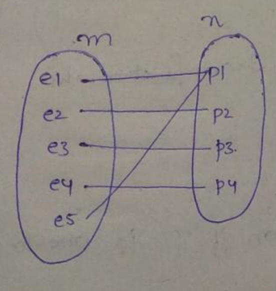
Ex: If $|A| = n$, $|B| = 2$ $(n>2)$ then the no. of onto functions from A to B?

$\Rightarrow 2^n - {}^2C_1 (1)^n = \boxed{2^n - 2}$

8. EXAMPLES ON ONTO FUNCTIONS - 2

In how many ways we can assign 5 employees to 4 projects, so that every employee is assigned to only one project and every project is assigned to atleast one employee?

Sol:



$m = 5 \quad n = 4$

Required ways: $4^5 - {}^4C_1 (3)^5 + {}^4C_2 (2)^5 - {}^4C_3 (1)^5 + 0$

$= 1024 - (243\times 4) + 6(32) + (-4) = \underline{\underline{240}}$

$\boxed{\underline{\underline{240}} \text{ ways.}}$

9. EXAMPLES ON ONTO FUNCTIONS - 3

Consider the following functions on set of all integers $f(x) = x^2, g(x) = x^3$ and $h(x) = \lceil x/2 \rceil$ which of the following is TRUE?

S1) f_1 is one-one (FALSE) S4) g is onto (FALSE)

S2) f is onto (FALSE) S5) h is one-one (TRUE)

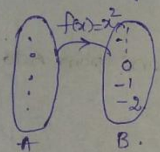
S3) g is one-one (TRUE) S6) h is onto

1>

$f(x) = x^2$

$\left.\begin{array}{l} f(1) = 1 \\ f(-1) = 1 \end{array}\right\}$ Two elements are mapped to same element so NOT ONE-ONE

2> Set of Integers $= \{-\alpha - - - 0 - - - \alpha\}$.

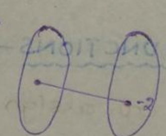


square of a number cannot be Negative but so Negative nos in B cannot be mapped with any element in 'A' not one-one.

3> $g(x) = x^3$

~~Not~~ one-one function

4>



Not pre Image for (-2) ∴ Not onto

5> $h(1) = \lceil 1/2 \rceil$

$h(1) = 1$
$h(2) = 1$ Not one-one function. and onto function also.

10. BIJECTION

A function $f: A \to B$ is called a Bijection if 'f' is one-to-one as well as onto.

→ If 'A' and 'B' are finite sets then Bijection from 'A' to 'B' is possible if $|A| = |B|$

→ If $|A| = |B| = n$ then No. of Bijections possible from A to B is $n!$

one-one
$f: A \to B$.
$|A| \le |B|$

onto function
$f: A \to B$.
$|A| \ge |B|$

$\underbrace{|A| = |B|}$ Bijection function condition.

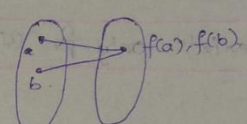
11. EXAMPLE ON BIJECTION

Let $A = R - \{3\}$ and $B = R - \{1\}$. A function $f: A \to B$ is defined by $f(x) = \frac{x-2}{x-3}$ which of the following is true?

a) f is one-one but not onto (X) (c) f is bijection (X)
b) f is onto but not one-one (d) f is neither one-one or onto.

1)

Let $f(a) = f(b)$



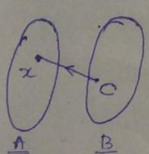
$= \frac{a-2}{a-3} = \frac{b-2}{b-3}$

$= (a-2)(b-3) = (b-2)(a-3)$

$= \cancel{ab} - 3a - 2b + \cancel{6} = \cancel{ab} - 3b - 2a + \cancel{6}$

$\Rightarrow \boxed{a = b}$ Not one to one

2)



$f(x) = c$

$\Rightarrow \frac{x-2}{x-3} = c$

$\Rightarrow x - 2 = cx - 3c$

$\Rightarrow x(1+c) = -3c + 2$

$\Rightarrow \boxed{x = \frac{2-3c}{1-c}}$ for every element 'c' we can find an element 'x' in 'A'.

12. INVERSE OF A FUNCTION

Let $f: A \to B$, if the inverse relation $f^{-1}: B \to A$ is a function then it is called inverse of function 'f' and it is ~~call~~ denoted by $f^{-1}: B \to A$

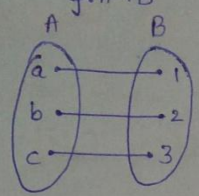
Theorem: Inverse of $f: A \to B$ exists iff 'f' is a bijection

Q) which of the following functions have inverse defined on their Ranges?
a) $f(x) = x^2$ b) $f(x) = x^3$.

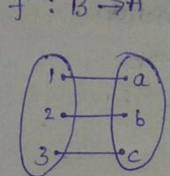
Sol)

$f: A \to B$

function.

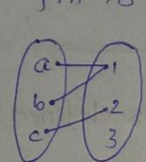


$f^{-1}: B \to A$



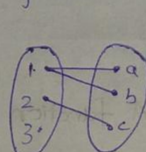
function $\Rightarrow f^{-1}$ exists.

$f: A \to B$



function

$f^{-1}: B \to A$



Not function

For a function to have Inverse it should be one-one function and onto function.

$\therefore$ $\boxed{\text{Inverse exists only for Bijection functions.}}$

a) $f(x) = x^2$

$\left.\begin{array}{l} f(1) = 1 \\ f(-1) = 1 \end{array}\right\}$ Not one-one $\Rightarrow$ Not Bijection.

b) $f(x) = x^3$. $\rightarrow$ one-one, onto, Bijection $\Rightarrow$ Inverse exists.

GRAPH THEORY

1. GRAPH THEORY-1

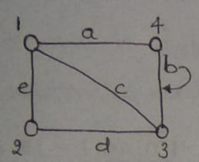
A Graph is represented as $G = (V, E)$

V = Vertices set = $\{v_1, v_2 --- v_n\}$

E = Edges set = $\{v_i \rightarrow v_j ---\}$.

Now, $|V|$ = No. of vertices in the Graph = order of the Graph

$|E|$ = No. of edges = size of the Graph.



$G = (V, E)$

$V = \{1, 2, 3, 4\}$

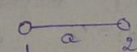
$E = \{a, b, c, d, e\}$

$a = \{1, 4\}$ $d = \{2, 3\}$

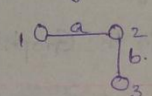
$b = \{4, 3\}$ $e = \{1, 2\}$

$c = \{1, 3\}$

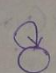
⇒ Adjacent vertices :- Vertices having common edge are adjacent



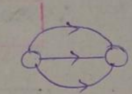
⇒ Advacent edges : If two vertices have an edge in common then the edges are called Adjacent edges.



⇒ Self-loop :



⇒ Multiedge :



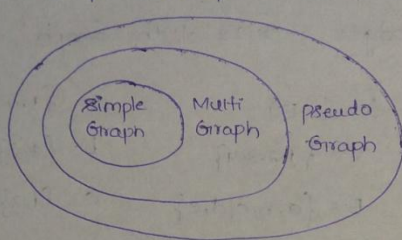
⇒ Simple Graph is the one which donot have self loops and multiedges.

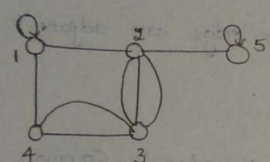
Graph	Self loops	Multiedges
General/pseudo Graph	✓	✓
MultiGraph	×	✓
Simple Graph	×	×

2. GRAPH THEORY- 2

Degree of the vertex : NO. of edges incident on it (a particular vertex) is called the degree of that vertex (counting loops twice)

⇒ Every simple Graph is a multigraph aswell as pseudo Graph. similarly a multiGraph is a pseudo Graph.





deg(1) = 4   deg(4) = (3)

deg(2) = 5   deg(5) = 3

deg(3) = 5

∴ sum of the degrees = 20.

No. of edges in the Graph = 10.

∴ $\boxed{\text{sum of the degrees} = 2(\text{No. of edges})}$ → Hand Shaking Lemma

∴ $\boxed{\sum_{v \in V} d(v) = 2|E|}$

2. The no. of vertices with odd degree in a Graph is always even.

$\sum deg(v) = 2|E|$

$\sum_{odd} deg(v) + \underbrace{\sum_{even} deg(v)}_{even} = even$

⇒ $\sum_{odd} deg(v) = even - even = even$

∴ $\boxed{\sum_{odd} deg(v) = even}$

∴ The no. of vertices with odd degree in a Graph is always even

3. GRAPH TH

Degree Seque

(Descending

Ex: d = 3

d = 3

⇒ Now, Giv

a deg se

or not

solution

HAVEL- H

① ⇒ put the

② ⇒ Remove

③ ⇒ Substrac

⇒ Repeat

Stop this p

⇒ we get

⇒ If we ge

⇒ Not eno

Ex:

1) (3, 2, 1, 1,

2)

3) (6, 5, 4,

⇒ 6 ↓

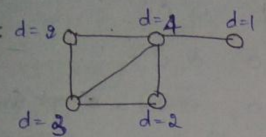
Now

sub

next

3. GRAPH THEORY-3

Degree Sequence: The Arrangement of degrees in Non-Ascending order (Descending order) is called the Degree sequence.

Ex:


Degree Sequence = (4, 3, 2, 2, 1)

⇒ Now, Given a graph finding the degree sequence is easy but given a deg sequence finding whether there exits atleast one simple Graph or not is difficult. we use "HAVEL-HAKIMI PROCEDURE" to find solution to such problem.

HAVEL-HAKIMI PROCEDURE

1. ⇒ put the degree sequence in Descending order
2. ⇒ Remove the highest degree (Let it be K)
3. ⇒ Substract one from the remaining K vertices

⇒ Repeat the steps 1 2 3

Stop this procedure when

⇒ we get all 0 entries ⇒ simple Graph exits

⇒ If we get Negative values ⇒ Simple Graph Not possible

⇒ Not enough edges ⇒ No Simple Graph.

Ex:

1) (3, 2, 1, 1, 0) ⇒ sum of degrees should be even ⇒ Here we get '7'

∴ No such Graph exists.

2) ($\cancel{3}$ 2 1 0 0)

(1 0 -1 0)

↳ Negative edge = No such Graph exists.

3) (6, 5, 4, 3, 3, 1)

⇒ $\cancel{6}$ 5 4 3 3 1

↓

Now cut '6' and substract '1' from the next 6 nodes but there are only 5 ⇒ so No Simple Graph Exists.

4) which of the following degree sequence doesnot correspond a Simple Graph

i) $(7,6,5,4,4,3,2,1) \Rightarrow (\cancel{7}\ 6\ 5\ 4\ 4\ 3\ 2\ 1)$ ④

$(\cancel{5}\ 4\ 3\ 3\ 2\ 1\ 0)$

$(\cancel{3}\ 2\ 2\ 1\ 0\ 0)$

$(\cancel{1}\ 1\ 0\ 0\ 0)$

$(0\ 0\ 0\ 0)$ = simple Graph exists.

2) $(6,6,6,6,3,3,2,2) = (\cancel{6}\ 6\ 6\ 6\ 3\ 3\ 2.\ 2)$

$(\cancel{5}\ 5\ 5\ 2\ 2\ 1\ 2)$

$(\cancel{4}\ 4\ 1\ 1\ 0\ 2)$

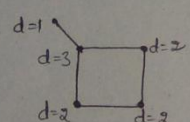
$(3\ 0\ 0\ -1\ 1)$

$\hookrightarrow$ (-ve) = No simple Graph

4. GRAPH THEORY-4

Min degree = "δ" = Min of all the degrees of the particular Graph

Max degree = "Δ" = Max of all the degress of the Graph is called Max deg.



$\boxed{\begin{matrix}\delta = 1 \\ \Delta = 3\end{matrix}}$

$\Rightarrow$ If 'G' is a graph with V- vertices and e- Edges then,

$$\boxed{\delta \leq \frac{2e}{V} \leq \Delta}$$

Proof:

$\sum \deg(v) = 2|E|$

$\Rightarrow$ Replacing each degree with min. degree

$\underbrace{\delta + \delta + \delta + \delta \dots + \delta}_{} \leq 2e$

$\Rightarrow$ V times

$\therefore \boxed{V \cdot \delta \leq 2e}$

$\Rightarrow$ Replacing with max degree we get

$= \Delta + \Delta + \Delta + \dots + \Delta \geq 2e$

$= \boxed{V \cdot \Delta \geq 2e} \Rightarrow \boxed{2e \leq V \cdot \Delta}$

$\therefore \quad V \cdot \delta \le 2e \le V \cdot \Delta$

$\Rightarrow \boxed{\delta \le \frac{2e}{V} \le \Delta}$

$\Rightarrow$ G is a Graph with 11 edges and min deg = 3 then what is the max no. of vertices ?

Now, $\delta \le \frac{2e}{V}$

$\Rightarrow 3 \le \frac{2 \times 11}{V} \Rightarrow V \le \frac{2 \times 11}{3}$

$\Rightarrow V \le 7.33$

$\Rightarrow \boxed{V = 7}$

$\Rightarrow$ G is a Graph with 12 vertices and max degree = 4, Then max no. of edges in the Graph G is ——?

$= 2e \le V \cdot \Delta$

$= 2e \le 12 * 4$

$= e \le 24 \Rightarrow \boxed{e = 24}$ (max no)

5. GRAPH THEORY 5

Special Graphs:

Null Graph: A graph with no edges and n-vertices is called Null Graph.

$N_1 =$ 0

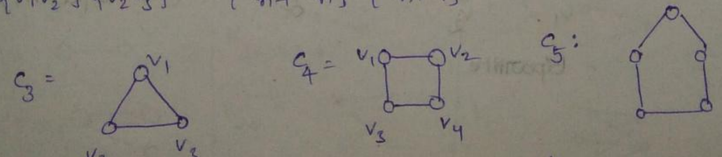
$N_2 =$ 0 0

$N_3 =$ 0 0 0

$N_4 =$ 0 0 0 0

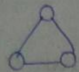
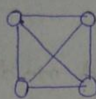
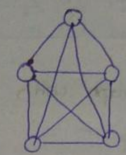
Cycle Graph

The cycle Graph is a simple Graph with n- vertices $\{v_1 v_2 --- v_n\}$ and edges $\{v_1 v_2\} \{v_2 v_3\} ---- \{v_{n-1} v_n\} \{v_n, v_1\}$



$\Rightarrow$ In cycle Graph 'C_n' deg of each vertex will be '2'. | No. of vertices = edges = n |

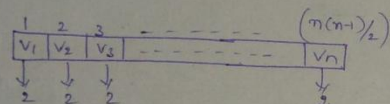
Complete Graph : complete Graph is a Simple Graph in which every pair of vertices are adjacent, Complete Graph denoted by 'K_n'. (6)

K_3 =  K_4 =  K_5 = 

	Vertices	Edges	deg(v)
K_n	n	$^nC_2$	$(n-1)$

⇒ Max no. of edges in simple Graph with 'n' edges = $^nC_2 = \frac{n(n-1)}{2}$

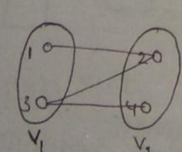
⇒ $V = \{v_1\ v_2\ v_3 \ ---- \ v_n\}$ of n- vertices then how many simple Graphs are possible?



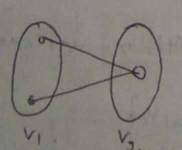
∴ $\boxed{\text{Total possible no. of simple graphs} = 2^{\left(\frac{n(n-1)}{2}\right)}}$

Bipartite Graphs :

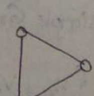
A Graph $G(V,E)$ is Bipartite Graph if the vertex set can be partitioned into two sets V_1 and V_2 such that every edge is in between vertex of V_1 and vertex of V_2.



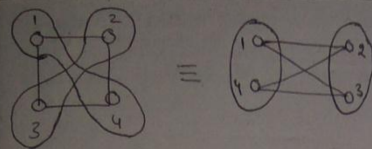
⇒ There should not be any edges between the vertices in a vertex set. (i.e. No edge between (1,3) and (2,4)



Bipartite



{ It is complete and cannot be Bipartite)



∴ The given graph is Bipartite Graph.

C_n is Bipartite if n = even	A complete Graph K_n can never be
C_n is not Bipartite if n = odd	Bi partite Graph.

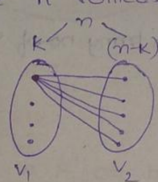
6. MAX NO. OF EDGES IN A COMPLETE BIPARTITE GRAPH

⇒ which of the following is the max no. of edges in Bipartite graph with n-vertices

(a) n^2 (b) $n^2/2$ ✓(c) $n^2/4$ (d) $n/2$

⇒ ~~Bipartite Graph with 6 vertices.~~

Now, let us assume the 'n' vertices are divide into 2 sets having K and $(n-K)$ vertices



⇒ To get the max. no. of edges then every vertex in V_1 should be matched with all the vertex in V_2

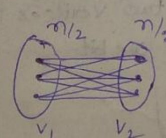
∴ The No. of edges = $K(n-K)$

∴ To get the max value of K, $\frac{d}{dK}(K(n-K)) = 0$

$= (n-K) + K(-1) = 0$

$\Rightarrow \boxed{K = n/2}$

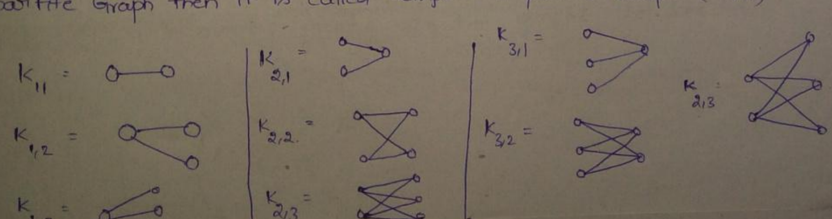
∴



⇒ No. of edges = $n/2 \times n/2 = n^2/4$.

7. COMPLETE BIPARTITE GRAPH AND REGULAR GRAPHS.

Complete Bipartite Graph: If we try to put every possible edge in the Bi-partite Graph then it is called Complete Bi-partite Graph ($K_{m,n}$)



	v	e	d(v)
K_{mn}	m+n	m*n	$\left\{\begin{matrix} m: v \in V_2 \\ n: v \in V_1 \end{matrix}\right\}$

$|V_1| = m$

$|V_2| = n.$

Regular Graph:

A Graph in which every vertex has same degree is called Regular graph

⇒ Every complete Graph (K_n) is always a Regular Graph.

Ex: $N_n = 0$ (0- Regular graph) ← Null Graph

$C_n =$ (2- Regular Graph)

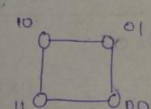
8. N- Cube

Now 1 cube means with 1 bits how many no-of vertices are possible $= 2^1 = 2$

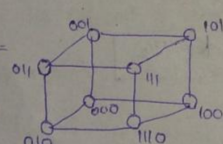
1- cube =

Now, 2cube means with 2-bits how many no-of vertices are possible $= 4 = 2^2$

2-cube =



similarly 3- cube =



{ An edge is present between two vertices if they differ by one bit.

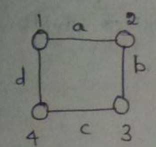
∴ $\boxed{\text{No. of Vertices in N-cube Graph} = (2)^N}$

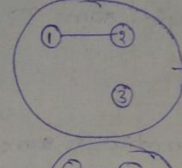
$\boxed{\text{Deg. of Each Vertex} = N}$

$\boxed{\text{No. of Edges} = N * 2^{N-1}}$

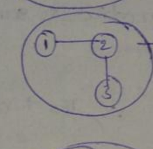
9. SUB-GRAPHS

A Graph $H(V',E')$ is a subgraph of $G(V,E)$ if $V' \subseteq V$, $E' \subseteq E$

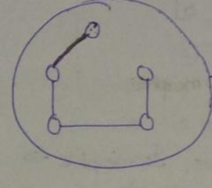




→ Sub Graph of given Graph



→ Induced sub graph (If i take all the possible edges between the choosen vertices that are given in the Actual Graph).



→ Spanning subGraph (If i take all the vertices of a Graph it is called spanning sub Graph. It may contain edges also.

10. ADJACENCY MATRIX

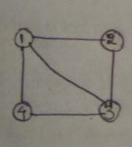
A Graph $G(V,E)$, $|V| = n$ it can be represented as $n \times n$ - matrix

$$A[G] = \begin{cases} 0 & \text{if } \{i,j\} \notin E \\ 1 & \text{if } \{i,j\} \in E \end{cases}$$

⇒ No provision for parallel edges.

⇒ Graph has no self loops iff the matrix diagnol entries are Zero's.

→ For Graph having no self loops $deg(v_i)$ = sum of entries in the i_{th} row.



$$A[G] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

→ deg of (1) = 3 (No. of 1's in that row)

→ d(2) = 2

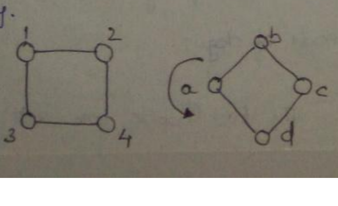
→ d(3) = 3

→ d(4) = 2

11. ISOMORPHISIM INTRODUCTION

Isomorphisim: Isomorphic graphs are the graphs that are drawn differently.

Ex:



The time complexity for checking whether the Graphs are Isomorphic or not is $O(n!)$

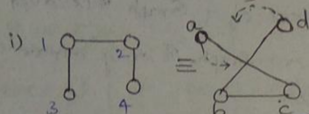
⇒ Isomorphic means two graphs should have same no. of vertices and their adjacencies should be preserved.

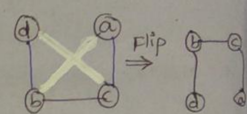
⇒ For the above examples. the adjacency matrix is same

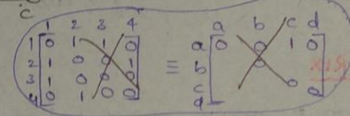
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 0 & 1 & 0 \end{array} \equiv \begin{array}{c|cccc} & & & & \\ \hline a & 0 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 1 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

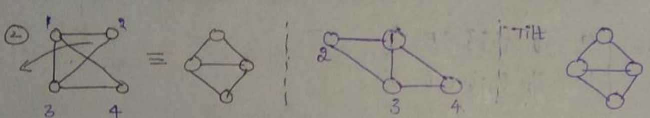
∴ The two graphs are Isomorphic

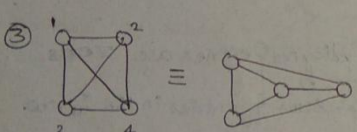
Which of the following Graphs are Isomorphic

i) 

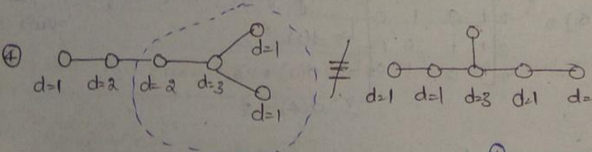
⇒ They are Isomorphic ⇒ 

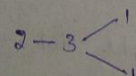


2) 

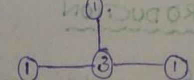
3) 

lift the node 4 and put in centre

4) 



∴ The node Having deg=3 is adjacent to nodes of deg=2 and deg=1, deg=1



Here the node of deg=3 is not adjacent to the node of deg=2

∴ The two Graphs are not Isomorphic

12. ISOMORP

Are these G

In this graph of deg=3 is nodes of de deg=3.

⇒ which of

13. SELF C

Complement

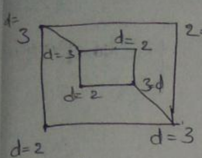
Let G(V,E

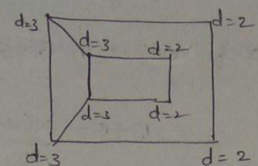
Such that

Ex

12. ISOMORPHISIM EXAMPLES

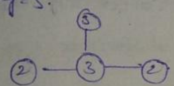
Are these Graphs Isomorphic?



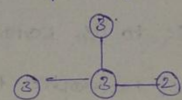


First check
1) No.of vertices and Edges
$|V|(G_1) = |V|(G_2)$
2) $|E|(G_1) = |E|(G_2)$
3) check for Adjacency

In this graph every node of deg=3 is adjacent to nodes of deg=2, deg=2, deg=3.

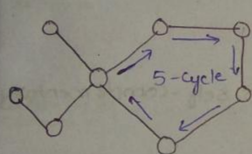


Every node of deg=3 is adjacent to nodes of deg=2, deg=3, deg=3.

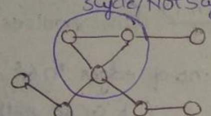


∴ There is a conflict ∴ These Graphs are not Isomorphic

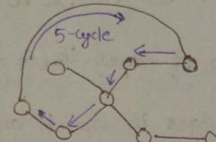
⇒ which of the graphs is Isomorphic to the Graph



(A) ×



(B)



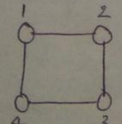
✓ And you can check degree sequence also.

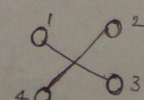
13. SELF COMPLEMENTING GRAPHS

Complement of a Graph

Let $G(V,E)$ be a simple Graph the the compliment of Graph $G = G^C = (V, E^C)$ such that two vertices are adjacent in G^C if they are not adjacent in G.

Ex





No.of edges in (G) + No of edges in $G' = {}^nC_2$ (∵ G, G' forms Complete Graph)

⇒ If the no. of edges in $G_1 = 13$ and No. of edges in $G_1^c = 15$ then the then what is the no. of vertices in G_1? (12)

⇒ we know that the no. of edges in $G_1 + G_1^c = \frac{N(N-1)}{2}$ [N = No. of vertices]

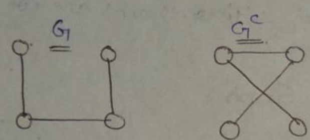
$= 13 + 15 = \frac{N(N-1)}{2}$

$= \boxed{N = 8}$

Self-complementing Graph

A graph which is isomorphic to its complement is called self-complementary graph. which means if G^c is isomorphic to 'G' then it is called Self Complement graph.

Ex:



$G_1^c \equiv G_1$ (Isomorphic Graphs)

⇒ which of the following cannot be the no. of vertices in self-complementary Graph. (A) 4 (B) 5 (C) 9 ~~(D)~~ 10

We know that the no. of edges in $(G_1 + G_1^c) = \frac{n(n-1)}{2}$ (n = vertices)

⇒ Let the no. of edges in $G_1 = K$ and so the no. of edges in $G_1^c = K$

$\therefore 2K = \frac{n(n-1)}{2}$ (∵ if 'G' is self complementary)

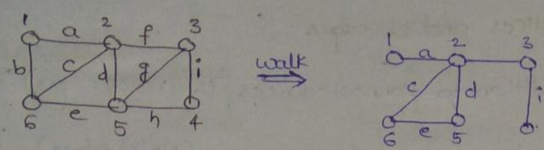
$\Rightarrow K = \frac{n(n-1)}{4}$ | Here K should be integer not fractions.

∴ n should be multiple of 4 or (n-1) should be ÷ble by 4.

14. CONNECTED COMPONENTS

Edge sequence: Sequence of edges starting and ending with vertex

Walk: Edge sequence in which no edge is Repeated (vertices can be repeated but not the edges).



(Vertext '2' is visited twice (vertex may be repeated but not the edge).

Closed walk: A walk in which start and end at same vertex is called closed walk otherwise it is open walk. (vertex may be repeated).

path: A path is an open walk in which no vertex is repeated.

cycle: A cycle is a closed walk in which no other vertex is repeated.

Results:

1. A graph is Bipartite if every cycle in the graph is even cycle.

Connected: Two vertices are said to be connected if there exists atleast one path between them.

⇒ The maximal connected subgraph is called "component".

Results:

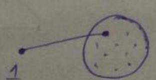
1) A simple Graph with n-vertices and k-components has atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

15. SUFFICIENT CONDITION FOR CONNECTEDNESS

The sufficient condition for connectedness is.

→ G is a simple Graph with n-vertices the (no. of edges) $> \frac{(n-1)(n-2)}{2}$

The 'G' is connected.



$(n-1)$ vertices

$(n-1)_{C_2}$ edges $= \frac{(n-1)(n-2)}{2}$.

⇒ which of the following Simple graphs are always connected.

a) 'G' with 5 vertices and 5 edges

b) 'G' with 6 vertices and 9 edges.

c) 'G' with 7 vertices and 13 edges.

d) ✓ 'G' with 8 vertices and 22 edges

⇒ The sufficient condition for connectedness is (no. of edges) $> {}^{(n-1)}C_2$

$$(\text{No of edges}) > \frac{(n-1)(n-2)}{2}$$

i) 5V, 5E ⇒ $5 > \frac{(4)(3)}{2}$ = $\boxed{5>6}$ false } Disconnected Graph. (Not connected)

ii) 6V, 9E ⇒ $9 > \frac{(5)(4)}{2}$ = $\boxed{9>10}$ = false } Disconnected.

iii) 7V, 13E ⇒ $13 > \frac{(6)(5)}{2}$ = $\boxed{13>15}$ false

iv) 8V, 22E: $22 > \frac{7\times 6}{2}$ $\boxed{22>21}$ } Connected

16. COMPLEMENTATION AND CONNECTEDNESS

Result: Atleast one of the Graphs 'G' or G^c is connected.

G	G^c	
C	C	possible cases.
D	C	possible cases.
C	D	possible cases.
D	D	Not possible

C = Connected
D = Disconnected

⇒ which of the following is always true?

S_1: G is connected, then G^c is disconnected

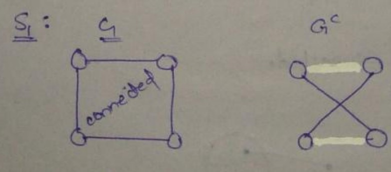
S_2: G is disconnected, then G^c is connected.

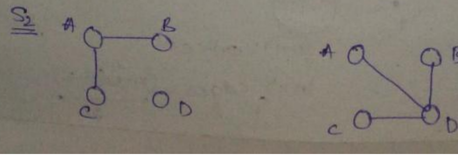
a) only S_1 is True (Not Necessarily true)

b) ✓ only S_2 is True

c) Both

d) None

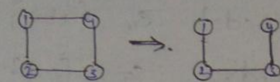




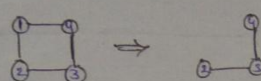
17. CUT VERTEX AND CUT EDGE

⇒ To make a Graph disconnected we may remove some edges or we may remove some vertices.

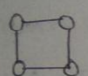
Removal of an edge :



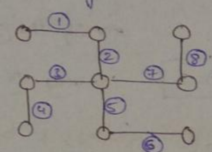
Removal of an vertex :



CUT EDGE (OR) BRIDGE : A single edge where removal of it makes the Graph dis-connected is called cutedge.



⇒ No cut edges are present



No. of cut edges = 8.

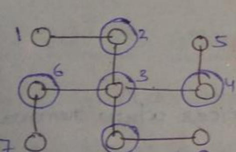
⇒ If there is a tree every edge is a cut edge

⇒ If there is no cycle then every edge is a cut edge

CUT VERTEX AND ARTICULATION POINT

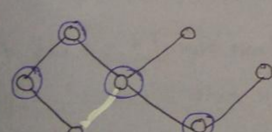
A single vertex whose removal disconnects the connected Graph is called cut vertex or "ARTICULATION POINT."

1)



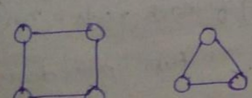
cut vertices / Articulation points are = {2, 3, 4, 6, 8}

2)



No. of cutvertices are (a) 3 ~~(b)~~ 4 (c) 5 (d) 6

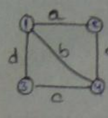
⇒ A Graph having no Articulation point / cut vertex is called a Biconnected Graph = All cycles are Bi-connected Graphs.



18. CUT SET AND EDGE CONNECTIVITY

Cut set: A set of edges where removal disconnects the Graph is called cutset. There can be more cutset for a given Graph.

Ex:



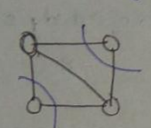
$c_1 = \{abcde\}$ $\quad$ $c_4 = \{abc\}$

$c_2 = \{abd\}$ $\quad$ $c_5 = \{cd\}$

$c_3 = \{ae\}$ → Minimum cut set

Edge-connectivity (λ):

The min. no. of edges where removal disconnects the graph G is called Edge-connectivity (λ).



$\boxed{\lambda = 2}$

⇒ If the Graph contains cycle then edge connectivity λ will always be greater than one $\boxed{\lambda > 1}$.

⇒ The edge connectivity is always upper bounded by min degree $\therefore \boxed{\lambda \le \delta}$

W.K.T $\quad V \cdot \delta \le 2E$, $\quad \delta \le \frac{2E}{V}$ $\Rightarrow$ $\boxed{\lambda \le \frac{2E}{V}}$

19. VERTEX CONNECTIVITY AND EDGE CONNECTIVITY

Vertex Connectivity (K = kappa): The min no. of vertices where removal disconnects the graph (or) leaves trivial graph is called Vertex Connectivity.

0 → Trivial Graph (K_1).

$$\boxed{K \le \lambda \le \delta \le \frac{2E}{V}}$$

G	λ	K
C_n	2	2
W_n	3	3
K_n	$(n-1)$	$(n-1)$
$K_{m,n}$	min (m,n)	min (m,n)

Wheel Graph W_n contains $(n+1)$ vertices

W_n has $2(n-1)$ edges

deg of each node = 3 (which are in cycle)

Hub vertex degree = $n-1$

20. WH
In any
⇒ A Gra
verte
Ac
⇒ A G
max val
Sol
21. EULE
⇒ If you
exact
walk
⇒ If you
all the
⇒ If you
. other
Graph
⇒ If a g
(The
Result: A

20. WHITNEY'S THEOREM

In any Graph $\boxed{K \le \lambda \le \delta}$

$\boxed{\text{Vertex Connectivity} \le \text{Edge Connectivity} \le \text{Mindegree}}$

⇒ A Graph with 11 edges and min degree is '4'. what is the max value of vertex connectivity?

Acc. to whitney theorem $K \le \lambda \le \delta \le \frac{2E}{V}$

⇒ $K \le \frac{2E}{V}$ | $\delta \le \frac{2E}{V}$

⇒ $K \le \frac{2 \times 11}{V}$ | $\delta \le$

⇒ $K \le \delta$

⇒ $\boxed{K \le 4}$

∴ max value $\boxed{K = 4}$

⇒ A Graph 'G' with 11 edges and 7 vertices is given what is the max value of K (vertex connectivity)?

Sol $K \le \frac{2E}{V}$

$K \le \frac{2 \times 11}{7}$

$K \le 3.1$ ⇒ max value $= 3$ ∴ $\boxed{K = 3}$

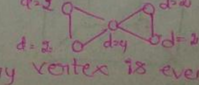
21. EULER'S GRAPH

⇒ If you can start from a vertex and if you can travel all edges exactly once and again comeback to the same vertex then such a walk is called Euler circuit and such a Graph is called Euler Graph.

⇒ If you start from a node and come back to the same node by covering all the vertices then it is called Eulerian circuit / Euler path

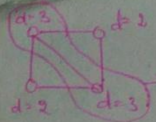
⇒ If you start from a node and cover all the nodes and reach some other node which is final then it is called unicursal path and the Graph is called unicursal Graph

⇒ If a graph has to be Euler Graph it should contain Evendegree. (The vertices should have Even degree.)



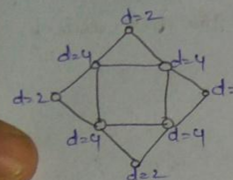
Result: A multigraph is Euler Graph iff degree of every vertex is even.

(18)



⇒ so Not Euler graph. so uni cursal Graph.

⇒ Is this Graph a Euler Graph



All degrees are Even ∴ The Graph is Euler Graph.

Result: A Multigraph is unicursal graph if there are exactly 2 vertices of odd degree.

22. HAMILTONIAN GRAPH (H- Graph)

→ In Euler Graph every edge should be covered and vertex can be repeated

→ In "Hamiltonian Graph" all the vertices should be covered and nothing should be Repeated (Need not cover all the edges)

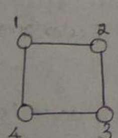
E-E (Euler Graph- should cover all the Edges).

⇒ You have to draw a cycle in the Graph such that it covers all the Vertices

⇒ A Graph containing Hamiltonian Cycle is called Hamiltonian-Graph.

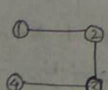
⇒ A path containing all the vertices of the Graph is called "Hamiltonian Path".

Eg:

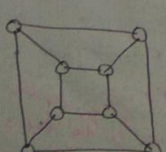


1-2-3-4-1 = Hamiltonian cycle

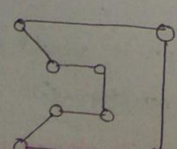
= Hamiltonian Graph (H-Graph)



= 1-2-3-4 = Hamiltonian path (Remove one edge from Hamiltonian-cycle then you get H-path).



≡



= Hamiltonian Cycle = Hamiltonian path

Results:

1) If 'G' is a Hamiltonian Graph with n-vertices

   1) Hamiltonian cycle, H-path contains n-vertices.
   2) Hamiltonian cycle contains n-edges.
   3) H-path contains $(n-1)$ edges.
   4) In a Hamiltonian cycle each vertex should have $(deg = 2)$.

The sufficient condition for Hamiltonian cycle is (Not Necessary)

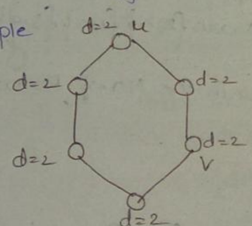
a) Diracs theorem: (simple Graph)

If min degree $\boxed{(\delta) \geq n/2}$ then G is Hamiltonian Graph

b) Ores Theorem: If $deg(u) + deg(v) \geq n$ (for $n \geq 3$) then for every pair of non adjacent vertices, then 'G' is H-Graph.

⇒ If a Graph satisfies Dirac, Ores theorem G is Hamiltonian but if a Graph doesnt satisfy above Theorems we cannot say 'G' is Non-H Graph.

For example



Here $\delta = 2$ $n = 6$

$\boxed{\delta \geq n/2} \Rightarrow 2 \geq 6/2 \Rightarrow (2 \geq 3)$ false

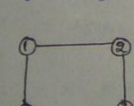
but the Graph is Hamiltonian.

$d(u) + d(v) = 4$ $(4 \geq n)$ false but 'G' is H-graph.

23. POWERS OF ADJACENCY MATRIX

Let $A[G]$ be the Adjacency matrix. then $[A(G)]^n$ Represents powers of Adjacency matrix.

Ex:



$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \Rightarrow A^2 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \end{array}$$

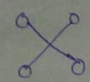
→ Represents the no. of paths of length 1 from 1 to 2

→ Represents the no. of paths of length 2 from 1 to 3. $\begin{cases} 1-2-3 \\ 1-4-3 \end{cases}$

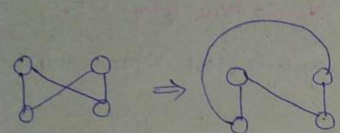
⇒ A^n Represents the no. of paths btn two vertices of length 'n'.

24. PLANAR GRAPHS

⇒ If we can draw a Graph on a paper without Crossovers then the Graph is planar. This concept is used in VLSI design.

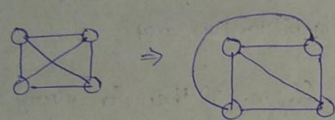
⇒ Cross over: 

Ex:



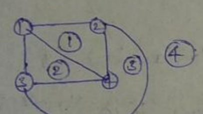
∴ The given graph is planar

K_4 =



⇒ ∴ K_4 is planar.

⇒



Internal Region = {1, 2, 3}

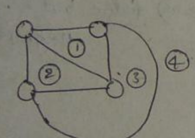
External Region = {4}.

The edge that divides two Regions is called Boundary edge.

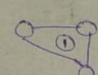
Here the Boundary edges are {2,4} {1,4} {3,4}

Degree of the Region: Degree of the Region is the No. of Boundary edges touching it.

Ex



Region	deg(R)
1	3
2	3
3	3
4	3.

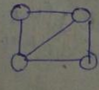
 = 3 edges.

⇒ sum of degrees of the Regions = Twice the no. of Boundary edges.

25. EULER FORMULA FOR SIMPLE GRAPH

Let 'G' be a connected planar graph with v - vertices, e = edges, r = regions then

$$\boxed{v - e + r = 2}$$



v = 4
e = 5
r = 3

v − e + r = 4 − 5 + 3
= 7 − 5 = 2//.

⇒ Min degree of a Region in simple Graph = 3.

⇒ Now, $\sum deg(r) = 2e$

$= r_1 + r_2 + r_3 + \dots + r_n = 2e$

$= 3+4+5+\dots+x = 2e$

$= 3+3+3+\dots+3 \le 2e$

$= 3|r| \le 2e \Rightarrow \boxed{3r \le 2e}$

$\hookrightarrow \boxed{r \le 2e/3}$ (or) $\boxed{e \ge \frac{3r}{2}}$

Now, Euler formula

$\Rightarrow v-e+r = 2$

$\Rightarrow v - e + \frac{2e}{3} \ge 2$

$\Rightarrow \boxed{e \le 3v-6}$

$3r \le 2e$

$\Rightarrow \boxed{r \le 2e/3}$

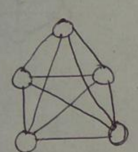
Now, $v-e+r=2$

$\Rightarrow v - \frac{3r}{2} + r \le 2$

$\Rightarrow \boxed{2v - r \le 4}$

26. K_5 is NOT PLANAR

$K_5 =$



K_5 = Kuratowski's Graph = $K_{3,3}$ is also called Kuratowski's Graph.

$v=5$

$e=10$

By Euler formula $v-e+r = 5-10+r=2$

$\Rightarrow \boxed{r=7}$

Now, $3r \le 2e$

$\Rightarrow 21 \le 2\times10$

$= \boxed{21 \le 20}$ false ∴ K_5 is Not planar / It must not be connected

⇒ Since K_5 is connected, K_5 is Not-planar Graph.

27. $K_{3,3}$ IS NOT PLANAR PART 1 AND PART 2

W.K.T. $v-e+r=2$, Now let us assume that the min.deg = k then.

W.K.T. sum of degree = 2e

$= k+k+k+\dots+k = 2e$

$= k|r| \le 2e \Rightarrow \boxed{kr \le 2e}$

$\Rightarrow \boxed{r \le 2e/k}$

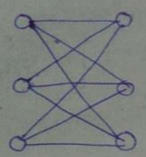
and $v-e+r = c+1$

$\hookrightarrow$ No. of connected components.

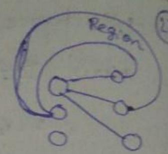
$\Rightarrow v - e + \frac{2e}{k} \ge 2 \Rightarrow \boxed{e \le \frac{k(v-2)}{k-2}}$ (or) $\boxed{v - \frac{kr}{2} + r \le 2}$

Now, let us check if K_{33} is planar or not

$K_{3,3}$:



Min. deg of any region = 4



Now, $v - e + r = 2$

$\Rightarrow 6 - 9 + r = 2$

$\boxed{r = 5}$ Now $e \leq \frac{4(4)}{2}$

$\boxed{9 \leq 8}$ - false

$\therefore K_{3,3}$ is not planar

Note: There wont be any Region of deg '3' in Bipartite Graph.

$\therefore$ The Graphs that contain K_{33} and K_5 as subgraphs are not planar.

1) The Graph which is Non-planar with min. no. of edges is 'K_5' $\therefore$ The Graphs with 2,3,4 vertices are planar. (simple Graphs)

2> $K_{3,3}$ is Non-planar with min. no. of edges $\therefore$ Any simple Graph with edges < 8. is planar.

so,

K_n is planar iff $n \leq 4$

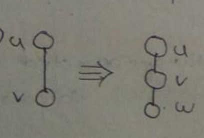
$K_{m,n}$ is planar iff $m \leq 2$ and $n \leq 2$

29. CHECKING FOR PLANARITY

Homomorphic operations

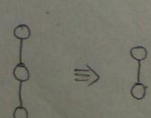
$\Rightarrow$ when ever you take a Graph and if we apply Homomorphic operations and we get another Graph then we can say that the two graphs are Homomorphic to each other.

Ex: 1)

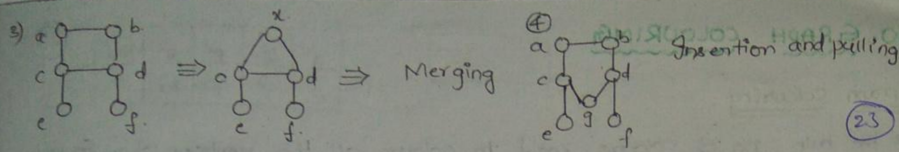


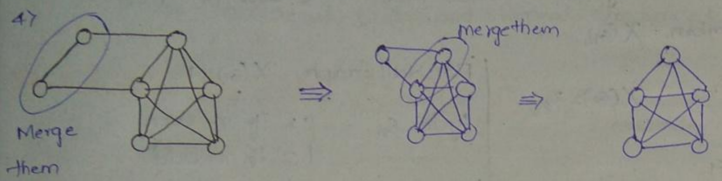
$\Rightarrow$ Insertion of vertex of $d=2$ in series.

2)



$\Rightarrow$ Removing a vertex

3) 

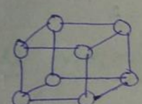
4) 

∴ The given Graph is Not planar.

Kuratowski's Theorem

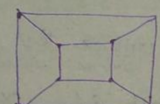
A connected graph is planar iff it is not homomorphic to K_5 or $K_{3,3}$.

Ex:



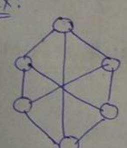
whether the Graph is Homomorphic or not?

⇒

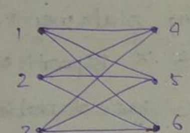


= planar (No cross overs)

Ex:



⇒



$= K_{3,3}$ ∴ NOT planar Graphs.

Ex:

1) $V = 20$, $\deg(v_i) = 3$ for all $v_i \in V$ then sum of degrees $= 2e = 60$ then what are the no. of Regions.

Sol: $V = 20$ sum of degree $\Rightarrow 2e = 60$ $\Rightarrow \boxed{e = 30}$

$d(v_i) = 3$.

| at min case each region will have min deg of '3'

$\therefore 3 + 3 + 3 + \cdots 3$ (20 times) ← each vertex degree

$= 60$

$= \boxed{2e}$

Now, $V - e + r = 2$

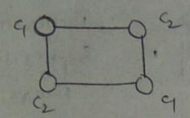
$= 20 - 30 + r = 2$

$= \boxed{r = 12}$

30. GRAPH COLOURING

Graph colouring:

⇒ The min no. of colours reqd to colour all the vertices of a Graph such that no two adjacent vertices have same color is called chromatic number. $\chi(G)$.



$\chi(G) = 2$

For Null Graph $\chi(G) = 1$

Cycle $C_n = \begin{cases} 2 & \text{if } n = \text{even} \\ 3 & \text{if } n = \text{odd} \end{cases}$

Wheel $W_n = \begin{cases} 3 & \text{if } n = \text{even} \\ 4 & \text{if } n = \text{odd} \end{cases}$

⇒ $K_n = \{n\}$

⇒ $K_{min} = 2$

Any Bipartite Graph, $\chi(G) = 2$

Properities

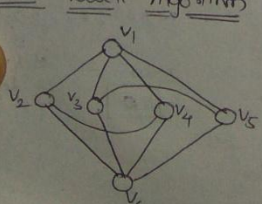
* 'G' is a graph with n-vertices then $\chi(G) \leq n$
* 'K_n' is a sub-graph of Graph 'G' $\chi(G) \geq n$
* $\chi(G) \leq 1 + \Delta$ → max degree
* $\chi(G) = \frac{|V|}{|V| - \delta}$

The following statements are Equivalent
1> 'G' is Bipartite
2> 'G' is 2-colorable
3> Every cycle in 'G' is Even cycle.

31. GRAPH COLOURING EXAMPLE-1

There is no Algo which can find the chromatic no. of a graph. (NP-Complete)
But we use some Greedy methods (Algo) like

Welsh-Powell-Algorithm



$d(v_1) = 4$
$d(v_6) = 4$

$d(v_2) = d(v_3) = d(v_4) = d(v_5) = 3$.

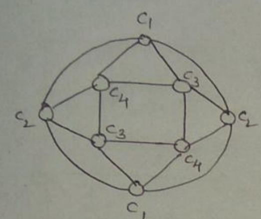
Now, sort the vertices in descending order of the degrees. = $v_1 v_6 v_2 v_3 v_4 v_5$

v_1	v_6	v_2	v_3	v_4	v_5
c_1	c_1	c_2	c_2	c_3	c_3

$\therefore \boxed{\chi(G) = 3.}$

⇒ start from 'v_1'. Now apply c_1 color to v_1 and the vertices that are not adjacent to it, and Repeat the procedure.

32. GRAPH - COLOURING EXAMPLE - 2

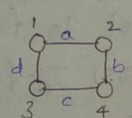


$\boxed{\chi(G) = 4}$

⇒ The Graph contains 'K_3' so the value of $\boxed{\chi(G) \geq 3.}$

33. MATCHING AND EDGE COVER

Matching: Set of Non-Adjacent edges.



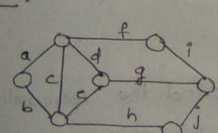
$M_1 = \{a\}$ $M_3 = \{b, d\}$

$M_2 = \{a, c\}$

Matching no: $[\alpha'(G)]$: Max no. of Non-Adjacent edges. For above Graph

$\boxed{\alpha'(G) = 2.}$

Ex:



$M_1 = \{a, i, h\}$

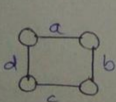
$M_2 = \{f, g, h\}$

$M_3 = \{i, e, a\}$

$\boxed{\alpha'(G) = 3}$

Edge cover:

What are all the edges that you should choose that covers all the vertices (Not isolated vertex)



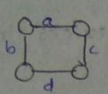
⇒ Edge cover = {abc} → {abcd}

= {bcd} → {ac}

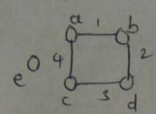
= {adc} → {db} ⇒

Edge covering NO: Min no. of edges that are Required to cover all the vertices is called Edge covering NO. $[\beta'(G)]$

Ex



$\Rightarrow \beta'(G) = \{2\} - \{ad\}$
$\{bc\}$



$\Rightarrow \beta'(G) = \{2\} + 1$ (for isolated vertex)

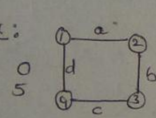
$\Rightarrow \boxed{\beta'(G) = 3}$

For any Graph $\boxed{\alpha'(G) + \beta'(G) = n}$

34. VERTEX COVER AND INDEPENDENT SET

Independent set: set of Non-Adjacent vertices is called Independent set

Ex:



Independent set $I_1 = \{1, 3\}$ $I_3 = \{1, 5\}$ $I_5 = \{3, 5\}$

$I_2 = \{2, 4\}$ $I_4 = \{2, 5\}$ $I_6 = \{4, 5\}$

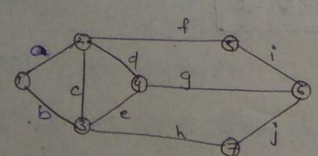
$I_7 = \{1, 3, 5\}$ $I_8 = \{2, 4, 5\}$

Independence no: $\alpha(G)$

Max No. of elements in the largest Independent set is called Max NO.

For the above graph $\alpha(G) = 3$ $[I_8, I_7]$

Ex:



$I_1 = \{1, 4, 5, 7\}$

$I_2 = \{2, 6\}$

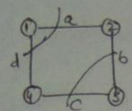
{Find the decreasing order of degree of vertices}

or

{Increasing order of vertices}

VERTEX COVER : set of vertices that cover all the edges is called vertex cover.

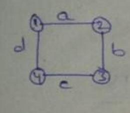
Ex:



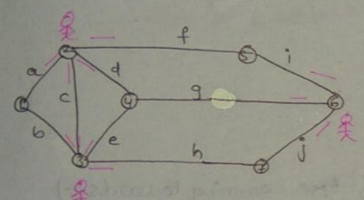
$V_1 = \{1,2,3,4\}$ $V_3 = \{2,4\}$

$V_2 = \{1,3\}$

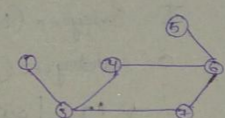
Vertex covering number : The min no. of vertices which can cover all the edges



$\beta(G) = 2$

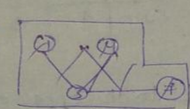


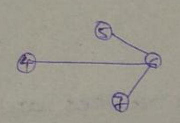
⇒ To get the vertex covering number remove the vertex with highest degree $= \{2,3\}$ so we can remove any one.



⇒ Now remove the vertex with next highest degree $= 6$ (In ~~main Graph~~ the Remaining Graph ~~not the Graph~~)

⇒





⇒ Now Remove $= 6$ then all vertices get Removed.

∴ we have removed three vertices ∴ $\beta(G) = 3$

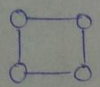
For any Graph "G" $\boxed{\alpha(G) + \beta(G) = n}$

Independence no + Vertex covering no = n.
Matching no + Edge covering no = n → $\alpha'(G) + \beta'(G) = n$

35. PERFECT MATCHING

A matching which can cover all the vertices of a Graph

Ex:



$M_1 = \{a, c\}$ can cover all vertices

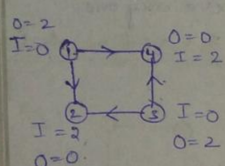
⇒ A graph 'G' has perfect matching only if 'G' has even no. of vertices.

*) No. of perfect matchings in $K_{2n} = \frac{(2n)!}{2^n (n!)}$

⇒ Find the no of perfect matchings in K_3 ⇒ $n = 3$ ⇒ $\frac{6!}{2^3\, 3!} = \underline{\underline{15}}$

⇒ No. of perfect matching in $K_{n,n} = n!$

36. DIRECTED GRAPH



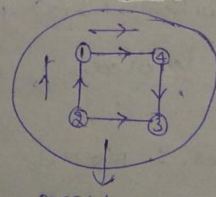
I = Indegree (No. of edges comming towards it)

O = outdegree (No. of edges outgoing from it)

V	In	out
1	0	2
2	2	0
3	0	2
4	2	0
sum	4	4

In any Graph sum of Indegrees = sum of outdegrees = No. of Edges

⇒ Two vertices are said to be connected if there exists a "directed path" between them, In the below Graph ② and ④ are connected.



→ This Graph is unilaterally connected because between every pair there exists a directed path.

possible pairs $= {}^4C_2 = 6$

(1,2)
(1,3)
(1,4)
(2,4)
(2,3)
(3,4)

= (1,2) (2,1) (3,1) ~~(4,1)~~
(1,3) ~~(2,3)~~ (3,2) (4,2)
(1,4) ~~(2,4)~~ ~~(3,4)~~ (4,3)

(2,3)

~~(1,4)~~
(1,2)

$\boxed{\begin{matrix} (2,1) & (1,4) \\ (2,4) & (1,3) \\ (2,3) & (4,3) \end{matrix}}$

COMBINATORICS

1. INTRODUCTION TO PERMUTATIONS AND COMBINATIONS

1. INTRODUCTION

$\Rightarrow n!$ (or) $\underline{|n}$

$\Rightarrow n! = 1 \cdot 2 \cdot 3 \cdots n$

$\Rightarrow 0! = 1$

} factorial of Negative numbers does not exist.

$\Rightarrow \boxed{n! = (n-1)! \times n}$

$\Rightarrow (n+1)! - n! \Rightarrow n!(n+1) - n!$

$\Rightarrow n!(n+\cancel{1}-\cancel{1})$

$\boxed{(n+1)! - n! = n*(n!)}$

$\Rightarrow (n-1)! + n! = (n-1)! + (n-1)! * n$

$= (n-1)!\,(1+n)$

$\boxed{(n-1)! + n! = (n+1)(n-1)!}$

$\Rightarrow {}^{n}P_{r} = \dfrac{n!}{(n-r)!}$

$\Rightarrow {}^{5}P_{3} = \underbrace{5 \times 4 \times 3}_{(3\text{ literals})} = 60$

$\Rightarrow {}^{10}P_{4} = 10 \times 9 \times 8 \times 7$

$= 5040.$

$\Rightarrow {}^{n}C_{r} = \dfrac{n!}{(n-r)!\,r!}$

$\Rightarrow \boxed{{}^{n}P_{r} = {}^{n}C_{r} \times r!}$

2. PROPERITIES OF NCR - 1

1) ${}^{n}C_{r} = {}^{n}C_{n-r}$ $\Rightarrow$ let $n-r = x$ $\Rightarrow {}^{n}C_{r} = {}^{n}C_{x}$

$\dfrac{n!}{(n-r)!\,r!} = \dfrac{n!}{(n-x)!\,x!}$

$= \dfrac{n!}{(n-(n-r))!\,(n-r)!}$

$\dfrac{n!}{(n-r)!\,r!} = \dfrac{n!}{r!\,(n-r)!}$ $\therefore \boxed{{}^{n}C_{r} = {}^{n}C_{n-r}}$

If ${}^{n}C_{x} = {}^{n}C_{y}$ then $\begin{cases} x = y \text{ or} \\ y = n-x \text{ or} \\ x = n-y. \end{cases}$

Now, consider n_{C_r}. Now the value of n_{C_r} is max iff

a) n is even $\Rightarrow \boxed{r = n/2}$

b) n is odd $\Rightarrow \boxed{r = \frac{n-1}{2} \text{ or } r = \frac{n+1}{2}}$

c) $n_{C_x} = n_{C_y} \Rightarrow \boxed{x+y=n}$

3. PROPERITIES OF NCR -- 2

$\Rightarrow \boxed{n_{C_r} + n_{C_{r-1}} = (n+1)_{C_r}}$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)!\times r} + \frac{n!}{(n-r)!\,(n-r+1)\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{n-\cancel{r}+1+\cancel{r}}{nr-r^2+r}\right]$$

$$= \frac{n!\,(n+1)}{(n-r+1)!\,r!} = \frac{(n+1)!}{(n+1-r)!\,r!} = (n+1)_{C_r}$$

Ex:

$10_{C_4} + 10_{C_3} = 11_{C_4}$

$= 10_{C_3} + 10_{C_4} = ?$ we think $r=3$ but × $r=4$

$(10_{C_4} + 10_{C_3}) = 11_{C_4}$

$10_{C_4} + 10_{C_3} = (11)_{C_{(4)}}$ — 1 more than 10; largest of 4, 3

4. PROPERITIES OF NCR-3

$$\Rightarrow n_{C_r} = \frac{n}{r}\,(n-1)_{C_{(r-1)}}$$

$$\Rightarrow n_{C_r} = \frac{n\,(n-1)!}{(n-r)!\,(r-1)!\,r}$$

$$= n/r\left[\frac{(n-1)!}{(n-r)!\,(r-1)!}\right]$$

$$= n/r\left[\frac{(n-1)!}{((n-1)-(r-1))!\,(r-1)!}\right] = n/r\left[(n-1)_{C_{(r-1)}}\right]$$

$$10_{C_3} = \frac{10!}{7!\,3!} \Rightarrow \frac{\cancel{7!}\times 8\times 9\times 10}{\cancel{7!}\times 3\times 2\times 1}$$

$$10_{C_3} = \frac{10}{3}\times 9_{C_3} = \frac{8\times 9\times 10}{3\times 2\times 1}$$

$$= \frac{10}{3}\times\frac{9}{2}\times 8_{C_1}$$

$$= \frac{10}{3}\times\frac{9}{2}\times\frac{8}{1}\times 7_{C_0} = \frac{10\times 9\times 8}{3\times 2\times 1}$$

5. PROPERTI

$\Rightarrow \frac{n_{C_r}}{n_{C_{r-1}}} =$

$\frac{n_{C_r}}{n_{C_{r-1}}} =$

$\frac{n_{C_r}}{n_{C_{r-1}}} =$

6. EXAMPLE

W.K.T =

7. EXAMP

$90_{C_9} + \sum_{r=0}^{10}$

$= 90_{C_9} + ($

$= n_{C_r} +$

$= 100_{C_8}$

5. PROPERITIES OF NCR – 4

$$\Rightarrow \boxed{\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{(n-r+1)}{r}}$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{\cancel{n!}}{(n-r)!\,r!} \times \frac{(n-r+1)!\,(r-1)!}{\cancel{n!}}$$

$$\boxed{\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{(n-r+1)}{r}}$$

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

$$\left.\begin{array}{l} {}^nC_0 + {}^nC_2 + {}^nC_4 + \dots + \dots = \\ {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots = \end{array}\right\} 2^{n-1}$$

$${}^{13}C_0 + {}^{13}C_1 + \dots + {}^{13}C_{13} = 2^{13}$$

$${}^{13}C_0 + {}^{13}C_2 + {}^{13}C_4 + \dots = 2^{12}$$

$${}^{13}C_1 + {}^{13}C_3 + {}^{13}C_5 + \dots = 2^{12}$$

6. EXAMPLE ONE NCR

W.K.T $= \dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{(n-r+1)}{r}$

Given ${}^nC_{r-1} = 36$, ${}^nC_r = 84$, ${}^nC_{r+1} = 126$

what is 'n' and r?

$$= \frac{84}{36} = \frac{n-r+1}{r}$$

$$= 7r = 3n - 3r + 3$$

$$= \boxed{10r = 3n+3} - (1)$$

Now, $\dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{n-(r+1)+1}{r+1}$

$$= \frac{n-r}{r+1} \therefore \boxed{\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}}$$

Now $\dfrac{n-r}{r+1} = \dfrac{126}{84} = \dfrac{3}{2}$

$$\Rightarrow 2n - 2r = 3r + 3$$

$$\Rightarrow \boxed{2n - 3 = 5r} - (2)$$

Solving (1) and (2) $\boxed{n=9}$ and $\boxed{r=3}$

7. EXAMPLE 2 ON NCR

$${}^{90}C_9 + \sum_{r=0}^{10} {}^{(100-r)}C_8 = ?$$

$$= {}^{90}C_9 + ({}^{100}C_8 + {}^{99}C_8 + {}^{98}C_8 + {}^{97}C_8 + {}^{96}C_8 + {}^{95}C_8 + {}^{94}C_8 + {}^{93}C_8 + {}^{92}C_8 + {}^{91}C_8 + {}^{90}C_8)$$

$$= {}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r \Rightarrow {}^{90}C_9 + {}^{90}C_8 = {}^{91}C_9 = {}^{92}C_9 = {}^{93}C_9 - {}^{94}C_9$$

$$= {}^{100}C_8 + {}^{100}C_9 = {}^{101}C_9$$

8. EXAMPLE 3 ON NCR

$^{50}C_{20} + 3\ ^{50}C_{21} + 3 \cdot {^{50}C_{22}} + {^{50}C_{23}}$

$= {^{50}C_{20}} + {^{50}C_{21}} + {^{50}C_{21}} + {^{50}C_{21}} + {^{50}C_{22}} + {^{50}C_{22}} + {^{50}C_{22}} + {^{50}C_{23}} + {^{50}C_{21}}$

$= {^{51}C_{21}} + {^{51}C_{22}} + {^{51}C_{22}} + {^{51}C_{22}} + {^{50}C_{23}}$

$\quad {^{52}C_{22}} + {^{51}C_{22}} + {^{51}C_{22}} + {^{50}C_{23}} \Rightarrow {^{51}C_{21}} + {^{51}C_{22}} + {^{51}C_{22}} + {^{50}C_{23}} + {^{51}C_{22}}$

This process doesn't work

$= {^{50}C_{20}} + 3\left({^{50}C_{21}} + {^{50}C_{22}}\right) + {^{50}C_{23}}$

$= {^{50}C_{20}} + 3\left({^{51}C_{22}}\right) + {^{50}C_{23}}$

$= {^{50}C_{20}} + {^{51}C_{22}} + {^{51}C_{22}} + {^{51}C_{22}} + {^{50}C_{23}}$

This combination doesn't work

$\Rightarrow \left({^{50}C_{20}} + {^{50}C_{21}}\right) + 2\left({^{50}C_{21}}\right) + 2\left({^{50}C_{22}}\right) + \left({^{50}C_{22}} + {^{50}C_{23}}\right)$

$\Rightarrow {^{51}C_{21}} + 2\left({^{50}C_{21}}\right) + 2\left({^{50}C_{22}}\right) + {^{51}C_{23}}$

$= {^{51}C_{21}} + 2\left({^{51}C_{22}}\right) + {^{51}C_{23}}$

$= \left({^{51}C_{21}} + {^{51}C_{22}}\right) + \left({^{51}C_{22}} + {^{51}C_{23}}\right)$

$= {^{52}C_{22}} + {^{52}C_{23}} = {^{53}C_{23}}$ //

9. EXAMPLE 4 ON NCR

1! + 4! + 7! + 10! + --- + 301! what is the last digit of the sum and 2 last digits of sum?

Sol: 1! + 4! + 7! + 10! + --- + 301!

⇒ Any thing after 10! will contain units digit as '0'. and '00' as last 2 digits.

⇒ Now consider 5! to 7! it also contains unit digit as '0' because 5! = 120

⇒ Now, the only thing that effects the sum is 1! + 4! = 25 //

∴ $\boxed{\text{last digit in sum} = 5}$

• 1 + 24 + 5040 = 5065 ∴ $\boxed{\text{last 2 digits} = 65}$ //

10. FUN D…

Assume t…

3 Red bal…

3 Black ba…

⇒ This pr…

2) Assum…

(a, b)

2 Shirts

2 pants

(1, 2)

- This pr…

⇒ (A)

⇒ 10 Bl…

10 Bla…

11. ROW…

⇒ perm…

⇒ choo…

⇒ The …

⇒ The …

10. FUNDAMENTAL RULES OF COUNTING

Assume there are

3 Red balls of diff sizes
3 Black balls of diff sizes

} The no. of ways you can choose a ball out of this 6 balls is '6' ways. This problem can be sub divided into two parts as

1) The no. of ways of choosing Red ball = 3w
2) The no. of ways of choosing Black ball = 3w
$\overline{6w}$

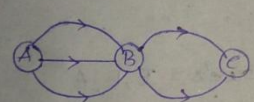
⇒ This principle / Rule is called Addition principle of probability.

2) Assume there are

(a, b)
2 Shirts
2 pants
(1, 2)

} The no. of ways you can dress up is (a,1) (a,2) (b,1) (b,2) } 4 ways = (2×2)

= This principle is called multiplication theorem on probability.

⇒ 

⇒ The no. of ways you can reach from 'A' to 'C' is 3×2 = 6 ways.

⇒ 10 Blue suits
10 Black suits

} The no. of ways you can dress up = 10+10 = 20 //.

11. ROW ARRANGEMENTS WITHOUT REPETETIONS

⇒ permutation = Arrangements

⇒ choosing = combination.

⇒ The no. of ways of arranging "abc" is abc, acb, bac, bca, cab, cba

⇒ $\boxed{\text{The no. of ways of arranging 'n' things, in a row is } n!}$

$\underset{n}{\_}\ \underset{(n-1)}{\_}\ \underset{(n-2)}{\_}\ \underset{(n-3)}{\_} \cdots \underset{1}{\_}$

$= 1 \cdot 2 \cdot 3 \cdots (n)$

$= n!$

12. EXAMPLES ON ROW ARRANGEMENTS WITHOUT REPETETIONS

1) CINEMA ⇒ The no. of letters/arrangements of this word = $6! = 720$

Restricted Permutations

⇒ The no. of words that are formed where the word always begin with 'C'.

∴ $\boxed{C}\ \underset{5}{\_}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}$ ∴ $1 \times 2 \times 3 \times 4 \times 5 = 120$ words.

⇒ Start with 'C' and end with 'A'.

$\boxed{C}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}\ \boxed{A}$ $= 4 \times 3 \times 2 \times 1 = 4! = 24.$

⇒ 'C' and 'A' should be the Extremes.

$\boxed{C}\ \underbrace{\underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}}_{4!}\ \boxed{A}$ & $\boxed{A}\ \underbrace{\_\ \_\ \_\ \_}_{4!}\ \boxed{C}$

$= 4! + 4!$

$= 48$ words.

⇒ Vowels are at even positions

1 2 3 4 5 6

3! ways

3! ways

Vowels = I, E, A

Consonants = C, N, M

$= 3! \times 3! = 36$ ways.

⇒ 'C' and I should always be together

$\boxed{C\ I}\ \underbrace{\_\ \_\ \_\ \_}_{4!\text{ ways}}$

5 ways

$= 5 \times 4! \times 2!$ Internal permutations of C, I

$= 5 \times 24 \times 2$

$= 10 \times 24$

$= 240$

(or) CINEMA

Single unit $= 5! \times 2!$

$= 240$

⇒ C,I,N should always be together

$$\Rightarrow \underbrace{\boxed{C|I|N}}_{\text{1 group}}\ \boxed{\ |\ |\ }$$

$4! \times 3! = 24 \times 6 = 124$

4 groups = 4! ways. ↳ Internal permutations

⇒ 'C' and 'I' should never be together. = Total Arrangements − CI come together

192 ⇐ $4 \times 2! \times 4!$	⇐	C | | I | | | |	
144 ⇐ $3 \times 2! \times 4!$	⇐	C | | | I | |	
96 ⇐ $2 \times 2! \times 4!$	⇐	C | | | | I |	
48 ⇐ $1 \times 2! \times 4!$	⇐	C | | | | | I	

480

$\Rightarrow \boxed{6! - (5! \times 2!)} = 480$

⇓

This procedure is applicable only for 2 letters not for 3 Alphabets.

⇒ C I N should always be together and 'I' should always be 'C' and 'N'.

C | I | N | | |

$2 \times 4 \times 1 \times 3! = 4 \times 6 = 24$ words $\times 2$ (C,N can be changed).

= 48 words

13. ROW ARRANGEMENTS WITH REPETETIONS

Arrangements with Repetetion. (some objects are identical)

The no. of ways of arranging 'n' elements in a row in which 'r' elements are identical is ~~called~~ = $\frac{n!}{r!}$

⇒ we have 'n' objects out of which 'm' are of one kind, 'r' are of another kind and 'p' objects are of other kind, then the no. of ways to arrange them is "$\frac{n!}{m!\, r!\, p!}$"

14. EXAMPLES ON ROW ARRANGEMENS WITH REPETETIONS - 1

RAVINDRABABU

R - 2 times

A - 3 times

B - 2 times.

⇒ NO of 12 lettered words that canbe formed is

$$\frac{12!}{2!\,3!\,2!}$$

Restricted Permutations ⇒ start with R

| R | | | | | | | | | | | |

$$\frac{11!}{3!\,2!}$$

⇒ start with 'R' and end with 'U'

| R | | | | | | | | | | | U |

$$\frac{10!}{3!\,2!}$$

⇒ Vowels and consonents should come together.

Vowels = A, I, A, A, U

consonants = R, V, N, D, R, B, B.

} 2 Groups 2! ways

$$2! \times \frac{5!}{3!} \times \frac{7!}{2!\,2!}$$

15. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS -- 2.

RAVINDRABABU.

R - 2 times

B - 2 times

A - 3 times.

⇒ All a's should come together

$\boxed{AAA}$ - - - - - - - - -

$$= \frac{10!}{\cancel{3!}\,2!\,2!} \times \frac{3!}{\cancel{3!}} = \frac{10!}{2!\,2!} = \begin{pmatrix} AAABB \\ \text{or} \\ AAA, B-B \end{pmatrix}$$

⇒ All A's should be together and B's should not be together

= $\boxed{AAA}$

= (All A's coming together) - (All B's not coming together).

$$= \left(\frac{10!}{2!\,2!} - \frac{9!}{2!} \times \frac{3!}{3!} \times \frac{2!}{2!} \right)$$ $\boxed{AAA}\boxed{BB}$ R, V, I, N, D, R, U

16. EXAMPLES ON

R's → not together

B's → should not be

Represen

together

on 'B'

∴ $\frac{12!}{2!\,3!\,2}$

∴ The N

17. EXAMPLE

2, 3, 4, 4, 5

18. EXAMPL

2, 3, 4, 4, 5.

⇒ Even dig

16. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS - 3

R's → not together

B's ⇒ should not be together

[Total methods] − [No. of Arrangements in which R, B come together) + (R's not together & B's come together) + (R together & B not together)

Total methods ↓

(RR, BB)
(R–R, BB)
(RR, B–B)
(R–R, B–B)

Represent R's come together & no Restriction on 'B'.

$$\therefore \frac{12!}{2!\,3!\,2!} - \left[\left[\frac{11!\times 2!}{2!\,3!\,2!}\right] + \left(\frac{11!}{3!\,2!} - \frac{10!}{3!}\right)\right]$$

→ Total − (R come together)

$$= \left(\frac{11!}{3!\times 2!} - \frac{10!}{3!}\right)$$

$$\therefore \text{The No. of ways} = \frac{12!}{2!\,3!\,2!} - \left[\frac{11!\times 2!}{2!\,3!\,2!} + \left(\frac{11!}{3!\,2!} - \frac{10!}{3!}\right)\right]$$

17. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS - 4

2, 3, 4, 4, 5

{ 2

⇒ Total no. of No's $= \frac{5!}{2!} = 60$

⇒ Even no's $=$ _ _ _ _ 2 (2, 4, 4) $\quad \frac{4!}{2!} = 12$ | _ _ _ _ (2, 4) $\quad \frac{4!}{1!} = 24$

⇒ 36 even no's.

= odd nos: _ _ _ _ 3 $\quad \frac{4!}{2!}$ | _ _ _ _ 5 $\quad \frac{4!}{2!}$

$$= 2\times\frac{4!}{2!} = 24 \text{ odd no's.}$$

18. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS - 5

2, 3, 4, 4, 5.

⇒ Even digits should be at odd places. (2, 4, 4)

$2!$

_ _ _ _ _

2 4 4 → $\frac{3!}{2!}$

$$\therefore \frac{3!}{2!}\times 2! = 6 \text{ ways.}$$

⇒ Find the no. of no's that are < 40,000

$\underset{2}{\underset{\sim}{2,3}}$ $\quad \dfrac{4!}{2!}$ $\quad = \dfrac{2 \times 4!}{2!} = 24$ words.

⇒ The no's should be < 50,000

2/ – Fix it then ⟶ Now, $\dfrac{4!}{2!}$ no's

3/ – Fix it then ⟶ Now, $4!/2!$ no's $\quad = 2\left(4!/2!\right) + 4!$

4 – Fix it then ⟶ Now, $4!$ no's. $\quad = 48$//.

19. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS – 6

2, 3, 4, 4, 5

→ Find the 5 digited no's which are ÷ble by 2. = 36 even no's.

⇒ ÷le by 3 ⇒ (A no' is ÷ble by 3 if the total sum is ÷ble by 3}

$2+3+4+4+5 = 18$

∴ The no's are $= \dfrac{5!}{2!}$

⇒ Divisible by 4 (last 2 digits should be ÷le by 4)

= _ _ _ _ _

_ _ _ (2,4) = $3!$

_ _ _ (4,4) = $3!$ $\quad$ ∴ The total no's = $3! + 3! + 2\left(3!/2!\right)$

_ _ _ (3,2) = $3!/2!$ $\quad = 3(3!)$

_ _ _ (5 2) = $3!/2!$ $\quad = 18$//

⇒ ÷le by 5

$\underbrace{- - - -}_{\frac{4!}{2!}}$ $\underline{\ 5\ }$

$\dfrac{4!}{2!} \times 1$ $\quad$ ∴ The no. of words = 12//

20. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS

0, 2, 3, 4, 5 ⇒ Find no. of 5 digited Number

⇒ $\square\square\square\square\square$ = (Total No's) − (No's having '0' in first place)

= $5! - 4!$

= $\underline{\underline{96}}$.

(or)

$\underbrace{\square}_{(2,3,4,5)\to 4}\underbrace{\square\square\square\square}_{4!}$ = $4 \times 4!$ no's

= 96 no's

⇒. Find the total no. of 5 digited no's that are even?

= $\square\square\square\square\underbrace{\square}_{(0,2,4) = 3\text{ chances}}$

Now, $\underbrace{\square\square\square\square}_{\text{No's} = 4!\text{ no's}}\boxed{0}$ $\quad$ $\underbrace{\square}_{(3,4,5)\to 3}\underbrace{\square\square\square}_{3!}\boxed{2}$ ∴ $\boxed{\text{no's} = 3 \times 3!}$

$\underbrace{\square}_{(2,3,5)\to 3}\underbrace{\square\square\square}_{3!}\boxed{4}$ = $3 \times 3!$ No's

∴ Total No's = $4! + (3 \times 3!) + (3 \times 3!)$

= $24 + 3(6) + 3(6)$

= $24 + 36$

= $\underline{\underline{60}}$.

⇒ How many 5 digited no's are present that are divisible by 5?

$\underbrace{\square\square\square\square}_{4!}\boxed{0}$ = $4!$ No's

$\underbrace{\square}_{(2,3,4)\to 3}\underbrace{\square\square\square}_{3!\text{ ways}}\boxed{5}$ = $3 \times 3!$ No's

⇒ $\boxed{\text{Total No's} = 4! + 3 * 3!}$

⇒ Find no. of 5 digited no's that are divisible by 4.

$\underbrace{\square\square\square}_{3!\text{ ways}}\ \underbrace{\square\square}_{6\text{ ways}}$

Given no's = 0,2,3,4,5

(20) (40)
(24) (52)
(32)
(04)

$\underbrace{\square\square\square}_{3!}\ \underbrace{\boxed{2}\boxed{0}}_{1} = 3!$

$\underbrace{\square\square\square}_{3!}\ \underbrace{\boxed{0}\boxed{4}}_{1} = 3!$ no's

$\underbrace{\square}_{2}\underbrace{\square\square}_{2!}\ \boxed{2}\boxed{4} = 2\times2!$

$\underbrace{\square\square\square}_{3!}\ \boxed{4}\boxed{0} = 3!$

$\underbrace{\square}_{(2)}\underbrace{\square\square}_{2!}\ \boxed{3}\boxed{2} = 2\times2!$

$\underbrace{\square}_{2}\underbrace{\square\square}_{2!}\ \boxed{5}\boxed{2} = 2\times2!$

∴ Total no's divisible by 4 = $3(3!) + 3(2\times2!)$
$= 3(6)+3(4)$
$= 18+12 = \underline{30}$

21. RANK IN A DICTIONARY

⇒ RAVI — 4! words can be formed. (Find the rank of RAVI)

⇒ A I R V — Alphabetical order

⇒ A _ _ _ = 3! words
I _ _ _ = 3! words
$\boxed{R}\ \boxed{A}$ I V = 1! word
R A V I = 1 word

14

⇒ RANK (Find the rank of the word Rank)

AKNR

A _ _ _ = 3!
K _ _ _ = 3!
N _ _ _ = 3!
~~R A~~
$\boxed{R}\boxed{A}$ K _ — 1! word
$\boxed{R}\boxed{A}\boxed{N}\boxed{K}$ — 1 word

20

∴ Rank of the word RANK = 20.

(12)

⇒ Find the Rank of sourav. = 6! words are formed

⇒ Alphabetical order = A O R S U V

(13)

A – 5!

O – 5!

R – 5!

$\boxed{S}$ A = 4!

$\boxed{S}\boxed{O}$ A = 3!

$\boxed{S}\boxed{O}$ R = 3!

$\boxed{S}\boxed{O}\boxed{U}$ A = 2!

$\boxed{S}\boxed{O}\boxed{U}\boxed{R}\boxed{A}\boxed{V}$ = 1

$\boxed{399}$

⇒ Find the Rank of IGNITE

Alphabetical order = E G I I N T

E – – – – – = $5!/2!$ = 60

G – – – – – = $5!/2!$ = 60

$\boxed{I}$ E – – – – = 4! = 24

$\boxed{I}\boxed{G}$ E – – – = 3! = 6

$\boxed{I}\boxed{G}$ I – – – = 3! = 6

$\boxed{I}\boxed{G}\boxed{N}$ E – – = 2! = 2

$\boxed{I}\boxed{G}\boxed{N}\boxed{I}$ E.T – 2! = 2

$\boxed{I}\boxed{G}\boxed{N}\boxed{I}\boxed{T}\boxed{E}$ = 1

160

22. RANK IN A DICTIONARY-2

⇒ Given the Alphabets A, I, R, V Find the word whoose rank is 16

Now find the word whoose rank is 14?

A! A _ _ _ = 3! = 6 (1-6)

I _ _ _ = 3! = (7-12)

R A _ _ = 2! (13/14)

RAIV — 13

RAVI = 14th

⇒ Find the word whoose rank is 20.

A _ _ _ = 3! = 6 (1-6)

I _ _ _ 3! = 6 (7-12)

R _ _ _ 3! = 6 (13-18)

V A _ _ = 2! (19,20)

VAIR - 19

VARI = 20

Rank = 18

A _ _ _ = 3! = 6

I _ _ _ = 3! = 6

R A _ _ = 2! = 2

R I _ _ = 2

R V _ _ = 2

R V A I = 17

R V IA = 18 ∴ RVIA = 18

⇒ Find the word whoose rank is 160. (E, G, I, I, N, T)

E _ _ _ _ _ = 5!/2! = 60 (1-60)

G _ _ _ _ _ = 5!/2! = 60 (61-120)

I E _ _ _ _ = 4! = 24 (121-144)

X I G _ _ _ _ = 4! = 24 (145 - 168) '160' is in the range so our Reqd word starts with IG.

I G E _ _ _ = 3! = 6 (145-150)

I G I _ _ _ 3! = 6 (151-156)

I G N _ _ _ = 3! = 6 (excees 160). ∴ The word starts with IGN

I G N E _ _ = 2! = (157-158)

I G N I T E = 160

23) CIRCULAR

⇒ The no. of ar

a, b, c

abc
acb
bac
bca
cab
cba.

24. CIRCULA

How many

a circle?

RAVINDRA

CIRCULAR

25. EXAMPL

A Family

how many w

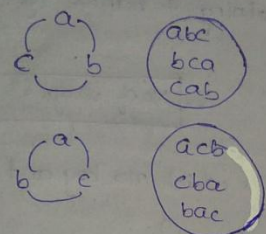
and Father

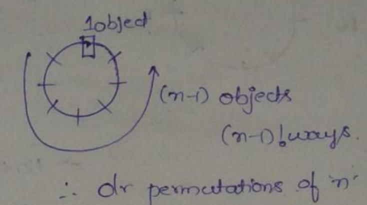
23) CIRCULAR PERMUTATIONS WITHOUT REPETETIONS.

⇒ The no. of arrangements of 'n' distinct objects around a circle is $(n-1)!$

a,b,c

abc
acb
bac
bca
cab
cba.





∴ cir permutations of 'n' distinct objects = $(n-1)!$

24. CIRCULAR PERMUTATIONS WITH REPETETIONS

How many ways you can arrange letters of the word RAVINDRA? In a circle?

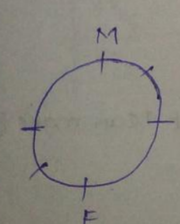
RAVINDRA = 8 letters ⇒ No. of Arrangements = $7!$ / A & R are repeated two times

$$\therefore \frac{7!}{2!\,2!}$$

CIRCULAR : $\frac{8!}{2!\,2!}$ PERMUTATIONS : $\frac{12!}{2!}$

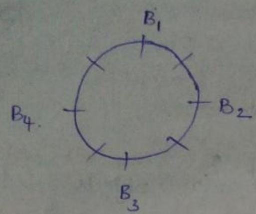
25. EXAMPLES ON CIRCULAR PERMUTATIONS - 1

A family has 6 mem, Mother, Father, sons ($S_1 S_2$), Daughters ($D_1 D_2$), Now how many ways you can make them sit in cir table such that Mother and Father faces each other.



Then 4 mem are remaining and 4 objects are present ∴ $4!$ ways.

2) We have 4 Boys $= B_1 B_2 B_3 B_4$, 4 Girls $- G_1 G_2 G_3 G_4$ } what are the no. of ways in which no two boys are together and no two girls are together



$\Rightarrow$ 4 boys can be arranged in circle in $3!$ ways

$\Rightarrow$ In the remaining 4 blanks 4 girls can be seated in $4!$ ways

$\therefore$ $\boxed{\text{No. of ways} = 4! \times 3!}$

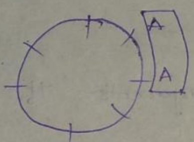
$\Rightarrow$ No. of ways in which no boy and Girl are together is $7! - (4! \times 3!)$

26. EXAMPLE ON CIRCULAR PERMUTATIONS -- 2

RAVINDRA $\Rightarrow$ Arrange in circle and ① Two a's always come together

$\Downarrow$

$\frac{8!}{2!\,2!}$ ways.



$= \frac{6!}{2!}$

$\underbrace{\text{R V I N D R}\boxed{\text{AA}}}_{\text{7 objects}}$

$6!$ ways

'R' is repeated two times.

$\Rightarrow$ Two a's should never come together.

$$\boxed{\text{No 2 a's come together} = \left(\frac{8!}{2!\,2!}\right) - \left(\frac{6!}{2!}\right)}$$

27. EXAMPLES ON GARLAND MODEL QUESTIONS

The no. of ways in which we can make garland from 'n' flowers is $\frac{(n-1)!}{2}$.

There are 5 flowers Now the no. of ways that I can make garland is $\frac{(n-1)!}{2} = \frac{(5-1)!}{2} = \frac{4!}{2} = \underline{\underline{12}}$

28. INTROD

The no. of w

is 'n_{C_r}' =

$3_{C_2} = 3 =$

29. EXAMP

10 Boys, 8 G

group sho

$= 10_{C_3} \times 8_{C}$

= Majority o

$\Rightarrow$ Majority o

$\Rightarrow$ We have

2 wicket

28. INTRODUCTION TO COMBINATIONS

The no. of ways of choosing 'r' objects from a set of 'n' distinct objects is 'n_{c_r}' $= \dfrac{n!}{(n-r)!\,r!}$

$3_{c_2} = 3$ = abc
- ab
- bc
- ac

} 3 ways.

$5_{c_2} = 10$ = abcde
- ab, ac, ad, ae
- bc, bd, be
- cd ce, de

29. EXAMPLE ON COMBINATION -- 1

10 Boys, 8 Girls. In how many ways you can choose 5 of them such that the group should contains 3 Boys and 2 Girls?

$= 10_{c_3} \times 8_{c_2}$

= Majority are Boys ⇒ (3 Boys + 2G) + (4B + 1G) + (5B, 0G)

$= (10_{c_3} + 8_{c_2}) + (10_{c_4} + 8_{c_1}) + (10_{c_5})$

⇒ Majority are Girls = (3G + 2B) + (4G + 1B) + (5G + 0B)

$= (8_{c_3} + 10_{c_2}) + (8_{c_4} + 10_{c_1}) + (8_{c_5})$

⇒ We have 22 players, form a team of 11, we have 12 Batsman, 8 Bowlers, 2 wicket Keepers, the team should contain (5 Batsmen, 5 bowlers, 1 WK) atleast.

BATSMAN (12)	BOWLERS (8)	[illegible] (2WK)	ways
12_{c_5}	8_{c_5}	2_{c_1}	$12_{c_5} \times 8_{c_5} \times 2_{c_1}$
12_{c_6}	8_{c_4}	2_{c_1}	$12_{c_6} \times 8_{c_4} \times 2_{c_1}$
12_{c_6}	8_{c_5}	1	$12_{c_6} \times 8_{c_5} \times 1$
⋮	⋮	⋮	⋮

30. EXAMPLE ON COMBINATIONS

⇒ A mathematics Question paper contains 12 Questions and we are supposed to ans 8 Then how many ways you can answer the Question paper? 12_{c_8}

⇒ The QP has 2 parts, the first 2 parts contains Questions from (1-6) and 2nd part contains Questions from (7-12). The Restriction is you have to answer atleast 3 from part 1 and In total you have to ans 8 Questions?

The no. of ways are,

(1 2 3 4 5 6) (7 8 9 10 11 12)

PART 1 PART 2.

$\Rightarrow 6_{c_3} \times 6_{c_5} =$

$\Rightarrow 6_{c_4} \times 6_{c_4} =$

$\Rightarrow 6_{c_5} \times 6_{c_3} =$

$\Rightarrow 6_{c_6} \times 6_{c_2} =$

The total ways are

$(6_{c_3} \times 6_{c_5}) + (6_{c_4} \times 6_{c_4}) + (6_{c_5} \times 6_{c_3}) + (6_{c_6} \times 6_{c_2})$

31. EXAMPLES ON COMBINATIONS -- 3

There are 10 couples (10m, 10w) Now, i want to form a team of 4 mem such that i have (2m and 2w) then how many ways we can form it.

No. of ways $\boxed{10_{c_2} \times 10_{c_2}}$

⇒ Now In the team of 4 people I should see that no couple is included.

⇒ $\boxed{10_{c_2} \times 8_{c_2}}$

⇒ Now, In the 4 mem team we need to include exactly one couple then

⇒ Now choose a couple from the 10 couples = 10_{c_1} (I have included 2 mem since i have chosen a couple)

⇒ Now, among the $\boxed{9m \text{ and } 9w}$

Choose 1 men = 9_{c_1}

⇒ Since I need to Include exactly one couple the I should not include the wife of the person choosen above, so im left with 8 mem, so choose 1 from them = 8_{c_1} ways.

∴ The tot

⇒ Now, In the 4

selected.

case 1: H_1

left

case 2: H_1

⇒ If H_1 is not se

may or maynot

ways =

∴ No. of

⇒ w_2, w_3 shou

W_2	W_3
✓	✓
✓	×
×	✓
×	×

32. EXAMPLES

⇒ There are 10

have shak

⇒ There are 2

persons =

$\therefore$ The total no. of ways $= \boxed{10_{C_1} \times 9_{C_1} \times 8_{C_1}}$

$\Rightarrow$ Now In the 4 member team, If H_1 is selected then W_5 should be selected.

case 1: H_1 is selected $\Rightarrow W_5$ should be included, Now, 9m, 9w are left $\Rightarrow$ No. of ways of selecting $= \boxed{9_{C_1} \times 9_{C_1}}$

case 2: H_1 is not included then, among 9 men choose 2 $= 9_{C_2}$ ways.

$\Rightarrow$ If H_1 is not selected it doesnt mean that you should not select W_5, W_5 may or may not be selected. $\Rightarrow 10_{C_2}$

$$\boxed{\text{ways} = 9_{C_2} \times 10_{C_2}}$$

$\therefore$ No. of ways $=$ case 1 $+$ case 2

$$\boxed{\text{ways} = 9_{C_1} \times 9_{C_1} + 9_{C_2} \times 10_{C_2}}$$

$\Rightarrow W_2, W_3$ should not be selected simultaneously.

W_2	W_3	
✓	✓	$= 10_{C_2}$
✓	×	$= 10_{C_2} \times 8_{C_1}$
×	✓	$= 10_{C_2} \times 8_{C_1}$
×	×	$= 10_{C_2} \times 8_{C_2}$

These are the cases that we need

$= 10_{C_2}(8_{C_1} + 8_{C_1} + 8_{C_2})$

32. EXAMPLES ON COMBINATIONS -- 4

$\Rightarrow$ There are 10 couples (10m, 10W) How many different ways we can have shakehands between them?

$\Rightarrow$ There are 20 people and we can have shakehands between any two persons $=$ $\therefore$ The total no. of shake hands $= 20_{C_2}$

$$= \frac{20 \times 19}{2} = 190 \text{ Shake Hands.}$$

⇒ Find the no. of shake hands that doesnot include a couple

No. of shakehands $= \left(20_{c_2} - 10\right)$ No. of shakehands of couples
10-couples = 10-shakehands

⇒ Let us say $H_1\ H_2\ H_3 - - - H_{10}$ and $w_1\ w_2\ w_3 - - - w_{10}$ } The condition is that there should not be hand shake between prime nos

$H_2\ H_3\ H_5\ H_7$
$w_2\ w_3\ w_5\ w_7$

There shouldnot be shakehands between these problems.

∴ The No. of shakehands $= \left(20_{c_2} - 8_{c_2}\right)$

33. REPETETIONS IN COMBINATIONS

aab $= 3!/2! = 3$ nos.

$- - - (3_{c_2} \times 1)$

$= (3 \text{ ways})$

aabcd $= 5!/2!$

$= 5_{c_2} \times 3!$ (take two objects and fix them aa and remaining objects can be placed in 3! ways.

There are 'n' numbers/objects in which 'r' are of similar kind then the no. of words that can be formed $= n!/r! = \left(n_{c_r}\right)\left(n-r\right)!$ → Arrange the Remaining ones
↳ choose the Repeating objects

34. ORDER

⇒ we have abc I have to form all 3 letterd words suchthat a should occur before B. ab, a_b. → 1 letter between a and b.
→ 'O' letter between ab.

a	b	c

3 positions

c a b —
a c b — } 3 ways.
a b c —

⇒ 1s choose 2 positions from available 3 positions $= 3_{c_2}$ (12 – abc, 13 – acb, 23 – cab) } 3

⇒ PERMUTATIONS (The condition is that vowels should always be in Alphabetical order

(21)

Vowels = (AEIOU) ← This is the order in which vowels should occur

⇒ Now choose 5 positions among 11 positions = $^{11}C_5$ ways (vowels cannot be permutated because order is specified here) and place the vowels.

⇒ Remaining '6' letters can be placed in $\frac{6!}{2!}$ ways ('T' Repeated 2 times)

$\therefore$ No of ways $= \frac{11!}{\cancel{6!}\,5!} \times \frac{\cancel{6!}}{2!}$

$$\boxed{\text{No of ways} = \frac{11!}{5!\,2!}}$$

⇒ We have 'n' different letters and we are supposed (we want) to find all 'n' lettered words in which 'r' letters should be in some order then no of words = $n!/r!$

35. EXAMPLE ON ORDER

RAVIND ⇒ The no of possible words that can be formed in which 'R' should always occur before A ?

⇒ Fix (R, A) ⇒ out of 6 positions choose any two = $^6C_2 \times 4!$ Remaining words.

$= \frac{6!}{2!\,4!} \times 4! = 6!/2!$

⇒ 'R' should come before 'A' and 'A' should come before V.

$R \to A \to V \Rightarrow {}^6C_3 \times 3!$ → Remaining words arrangements.

⇒ R should come before A and V → $(R \to A \to V) - 6!/3!$ or $(R \to V \to A) - \frac{6!}{3!}$

~~R→A→V 6!/3!~~

R → 4!

4 ways $\times\, {}^4C_2 \times 2!$

$\therefore\ 6!/3! + 6!/3! =$ ways.

36. EXAMPLES ON ORDER-2

RAVIND ⇒ Find All words such that 'R' occurs before A or 'R' occurs before 'B'?

⇒ Find the total words and substract the ways in which 'R' and V occur before R. ($A \to V \to R$, $V \to A \to R$)

$$= 6! - \left(\left(6_{C_3} \times 3! \right) + \left(6_{C_3} \times 3! \right) \right)$$

37. SELECTIONS WITH REPETETIONS

$abcd = 4_{C_2} = 6$

= ab
ac
ad
bc
bd
cd

$aabc = 4_{C_2}$ ~~$4!/2!$~~

$\left.\begin{array}{l} aa \\ ab \\ ac \\ bc \end{array}\right\} 4 = 4_{C_2}$

38. EXAMPLE ON SELECTIONS WITH REPETETIONS

⇒ aabc ⇒ Find/choose two objects

Case1: Two letters are same – oneway = 2_{C_2}

Case2: Two letters are different = 3_{C_2} = 3ways = (a, b, c)

} Total = 4 ways

⇒ RAVINDR ⇒ Choose two letters from this name

R–2
A–1
V–1
I–1
N–1
D–1

case 1: Both the letters are same = 1 way.

case 2: Both the letters are different = 6_{C_2}.

} Total = 6_{C_2} + 1 way

⇒ choose 3 letters from the given letters.

Case 1: 3 different letters = 6_{C_3}.

Case 2: '2' same + 1 different = $1 * 5_{C_1}$

} = $(6_{C_3} + 5)$ ways

⇒ RAVINDRA ⇒ choose two

1) 2 same – 2

2) 2 different – 6_{C_2}

} Total = $(2 + 6_{C_2})$

$\boxed{n_{P_r} = n_{C_r} \times r!}$

1. INTRODUCTI

$T(n) = T(n-1) +$

$T(n) = aT(n/b)$

2. RECURRANC

Sequence: Any

T(n)

n

T(0)

Recurrance Rela

present the

of previous

1).

2). Seq: 1, 3,

n = 0, 1,

T(0) = 1, T

T(n) =

$\boxed{{}^{n}P_{r} = {}^{n}C_{r} \times r!}$

2. RECURRANCE RELATIONS

1. INTRODUCTION

$T(n) = T(n-1) + T(n-2) + (T(n-3)) + K.$

$T(n) = aT(n/b) + K \rightarrow$ masters theorem

2. RECURRANCE RELATION AND SOLUTION

Sequence: Any order in which you write a number.

$T(n) = 0, 2, 4, 6, 8, 10 \ldots$

$n = 0, 1, 2, 3, 4, 5$

$T(0)=0, \; T(1)=2, \; T(2)=4, \; T(3)=6.$

Recurrance Relation: It is a relation, in which we are possible to write present the nth term / Any term in the sequence as an mathematic Expression of previous terms then it is called RR.

1)

$\boxed{T(n) = T(n-1) + 2}$ $(n \geq 2)$ [For above example)]

$T(0) = 0.$, $T(1) = 2$ } — Initial conditions

The main aim of topic RR is to find such sol to the RR's.

sol to above RR $T(n) = 2n$.

LHS of RR $= T(n) = 2n$

RHS of RR $= T(n-1) + 2 = 2(n-1) + 2$

$= 2n.$ //.

2) Seq: 1, 3, 5, 7, 9, 11 ...

$n = 0, 1, 2, 3, 4, 5$

$T(0) = 1, \; T(1) = 3, \; T(2) = 5$ $\boxed{T(n-1) + 2 = T(n)}$

$T(n) = 2n+1$

LHS of RR $= T(n) = 2n+1$

RHS of RR $= T(n-1) + 2$

$= \cancel{T[(2n-1)-1]+2}\; 2(n-1)+1+2$

$= 2n+1$ //.

$\boxed{T(n) = 2n+1}$ $n \geq 1$

$T(0) = 1$ (Initial condition)

Sequence: 1, 2, 4, 8, 16 - - - -

n: 0, 1, 2, 3, 4

T(0) = 1, T(1) = 2, T(2) = 4, T(3) = 8 - -

$\boxed{T(n) = 2T(n-1) ; n>0}$

T(n) = 0, n = 0 → [Initial condition]

Sol: $T(n) = 2^n$

LHS = $T(n) = 2^n$

RHS = $2T(n-1)$

$= 2^1 2^{n-1}$

$= 2^n$ LHS = RHS

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3. Examples

For the recurrance relation $T(n) = 2T(n-1) - T(n-2)$, $n \geq 2$, Find whether the following are solutions?

a) $T(n) = 3n$ b) $T(n) = 2^n$ c) $T(n) = 5$.

opvm,

a) $T(n) = 3n$.

$T(n) = 2T(n-1) - T(n-2)$

LHS = $T(n) = 3n$

RHS = $2T(n-1) - T(n-2)$

$= 2[3(n-1)] - [3(n-2)]$

$= 6(n-1) - [3n-6]$

$= 6n - \cancel{6} - 3n + \cancel{6} = 3n$. ✓ This is soln

(or)

T(n) = 3n

T(0) = 0

T(1) = 3

T(2) = 6

T(3) = 9

T(2) = 2T(1) - T(0)

= 2×3 - 0

= 6 ✓

T(3) = 2[T(2) - T(1)]

= 2(6) - 3

= 12 - 3 = 9.

b) $T(n) = 2^n$

T(0) = 1

T(1) = 2

T(2) = 4

T(3) = 8

T(2) = 2 T(1) - T(0)

= 2(2) - 1

3

Not equal

∴ 2^n is not solution.

c) T(n) = 5

T(0) = 5

T(1) = 5

T(2) = 5

T(3) = 5

T(2) = 2 T(1) - T(0) = (2×5) - 5 = 5

T(3) = T(3) = 2T(2) - T(1)

= 2×5 - 5 = 5

4. GATE 2004 QUES

The Recurrance relat

a) $2^{n+1} - n - 2$ b) 2^n

T(1) = 1

opA: $2^{1+1} - 1 - 2 =$

opB: $2^1 - 1 = 1$

T(2) = 2T(1) + 2 ⇒

opA: $2^3 - 2 - 2 = 4$

opB: $2^2 - 2 = 2$

5. GATE 2002 QU

The solution to th

a) 2^k b) $(3^{k+1} - 1$

T(1) = 1

a) Now if the

in Bigoh notat

opvm: - T(

T(1) = 1

4. GATE 2004 QUESTION

The Recurrance relation $T(n) = 2T(n-1)+n,\ n\geq 2$ Evaluates to

$T(1)=1$

a) $2^{n+1}-n-2$ b) 2^n-n c) $2^{n+1}-2n-2$ d) 2^n+n.

$T(1)=1$

opA:- $2^{1+1}-1-2 = 1$ c) $2^2-2-2=0$ ✗

opB: $2^1-1=1$ d) $2^n+n = 2^1+1 = 3$ ✗

$T(2) = 2T(1)+2 \Rightarrow 2(1)+2 = 4$

opA: $2^3-2-2=4$ ✓

opB: $2^2-2=2$

$\therefore$ $\boxed{\text{opA: } 2^{n+1}-n-2}$

5. GATE 2002 QUESTION

The solution to the Recurrance Relation $T(2^K) = 3T(2^{K-1})+1$

$T(1)=1$ is

a) 2^K b) $(3^{K+1}-1)/2$ c) $3^{\log_2 K}$ d) $2^{\log_3 K}$

$T(1)=1$

Q) Now if the question is $T(2^K) = 3T(2^{K-1})+1$ and options are given in Bigoh notations then how to solve them is.

$T(2^K) = 3T(2^{K-1})+1 \Rightarrow$ let $2^K = n$

$2^K/2 = n/2 \Rightarrow \boxed{2^{K-1} = n/2}$

$T(n) = 3T(n/2)+1 \longrightarrow T(n) = aT(n/b)+\Theta(n^K \log^P n)$

$a=3, b=2, K=0, P=0.$

$a>b^K \Rightarrow T(n) = \Theta(n^{\log_b a})$

$\Rightarrow T(n) = \Theta(n^{\log_2 3})$

$= \Theta((2^K)^{\log_2 3}) = \Theta(2^{K \cdot \log_2 3}) = \Theta(2^{\log_2 3^K})$

$\boxed{T(n) = \Theta(3^K)}$.

Given:- $T(2^K) = (3^{K+1}-1)/2$.

$T(1)=1 \Rightarrow$ put $K=0$ in $T=1$

$= (3^1-1)/2 = 1$ ✓

~~LHS: $T(2^K) = (3^{K+1}-1)/2$~~

~~RHS: $3T(2^{K-1})+1$~~ $\frac{\frac{3}{2}\left(\frac{3^{K+1}-1}{1}\right)}{2} = \frac{3}{4}($

~~$= 3\left[\left(\frac{3^{K+1}-1}{1}\right)/2\right]/2 =$~~

Now, if the questions are given of this form $T(n) = 3T(\sqrt{n}) + 1$ then convert it as

$T(n) = 3T(\sqrt{n}) + 1$ (24)

let $n = 2^k \Rightarrow T(2^k) = 3T(2^{k/2}) + 1$

$\Rightarrow$ let $2^k = p$ then $\boxed{T(p) = 3T(p/2) + 1}$

↳ Apply master theorem

6. GATE 2008 QUESTION

When $n = 2^{2k}$ for some $k \geq 0$, the reccurrance relation $T(n) = \sqrt{2}\,T(n/2) + \sqrt{n}$ evaluates to

$T(1) = 1$

a) $\sqrt{n}(\log n + 1)$ b) $\sqrt{n}\log n$ c) $\sqrt{n}\log\sqrt{n}$ d) $n\log\sqrt{n}$

$T(1) = 1 \Rightarrow T(2) = \sqrt{2}\,T(1) + \sqrt{2}$

$= 2\sqrt{2}$

opA: $\sqrt{n}(\log n + 1) = T(n)$

$T(1) = \sqrt{1}(\log 1 + 1) = 1$ ✓

$T(2) = \sqrt{2}(\log 2 + 1)$

$= \sqrt{2}(2) = 2\sqrt{2}$ ✓

8. GATE 2008 QUESTION

Let x_n denote the no. of binary strings of length 'n' that contain no consecutive 0's. Which of the following reccurrance relation does x_n satisfy

a) $x_n = 2x_{n-1}$ ~~b)~~ $x_n = x_{n/2} + 1$ c) $x_n = x_{n/2} + n$ ~~d)~~ $x_n = x_{n-1} + x_{n-2}$

opvm

x_1 = The no. of Binary strings that contain no consecutive ones $= \{1, 0\}$

$\Rightarrow x_1 = 2$

opa: $x_1 = 2x_0 = 0$ ✗

opB: $x_2 = x_{2/2} + 1 = 2 + 1 = 3$ ✗

opc: $x_2 = x_{2/2} + 1 = x_2 = 2 + 2 = 3 + 1 = 4$ ✗

opD: $x_2 = x_{one} + x_0 = 2 + 0 = 2.(3)$ ✗

$x_2 = \{01, 10, 11\}$

$= 3$

$x_3 = \{5\}$

op.D: $x_3 = x_2 + x_1$

$= 3 + 2$

$= 5$

9. GATE 2015 QUESTION

1

Let a_n Represent the no. of bit strings of length 'n' containing 2 consecutive 1's what is the recurrence relation for a_n?

a> $a_{n-2} + a_{n-1} + 2^{n-2}$ c> $a_{n-2} + 2a_{n-1} + 2^{n-2}$

b> $2a_{n-2} + a_{n-1} + 2^{n-2}$ d> $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

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$a_1 = 0$

$a_2 = 1$ {11}

$a_3 = 3 = \{011, 110, 111\}$

$a_4 = 8 = \{--\}$

opA : $a_3 = 8$ (from side Analysis)

$a_n = a_{n-2} + a_{n-1} + 2^{n-2}$

$a_3 = a_1 + a_2 + 2 = 0+1+2 = 3.$ ✓

opB : $a_1 + 2a_2 + 2 = 4$ ✗

opc : $2a_1 + a_2 + 2^1 = 3$ ✓

opd : $a_3 = 4$ ✗

Now, $a_4 = 8$ — opA : $a_2 + a_3 + 2^2 = 8$ ✓ $\therefore$ $\boxed{\text{opA}}$

— opc : $2\times1+3+4 = 9$

3. GENERATING FUNCTIONS

1. INTRODUCTION

Let us say we have a series $a_1 a_2 a_3 a_4 a_5$ now what is the generating function for the sequence. GF is nothing but we take each ele in the sequence and assign some weights. Now, the GF for the above sequence will be $a_1x^0 + a_2x^1 + a_3x^2 + a_4x^3 + a_5x^4$

Now, The seq: 1,1,1,1,1,1

GF: $1x^0 + 1x^1 + 1x^2 + 1x^3 + 1x^4 + x^5$

$= 1 + x + x^2 + x^3 + x^4 + x^5.$

Seq:- 1,1,1,1 - - -

GF $= 1\cdot x^0 + 1\cdot x^1 + 1\cdot x^2 + 1\cdot x^3 + ---$

GF $= 1 + x + x^2 + x^3 + ----$

2. EXAMPLE - 1

1> $1\cdot x^0 + 1\cdot x^1 = 1+x = \frac{(x^2-1)}{(x-1)} = \left[\frac{1-x^2}{1-x}\right]$ Given seq = 1,1

2> Seq: 1,1,1 $= 1+x+x^2 = 1+x+x^2 * \frac{(x-1)}{(x-1)} = \frac{(x^3-1)}{(x-1)} = \left[\frac{1-x^3}{1-x}\right]$

$1\cdot x^0 + 1\cdot x^1 + 1\cdot x^2$

3> 1, 1, 1 - - - n 1's

$$GF: 1 + x + x^2 + \cdots + x^n = \frac{x^n - 1}{x - 1} = \left[\frac{1 - x^n}{1 - x}\right]$$

4. INTRODUCTION TO PROPOSITIONAL CALCULUS.

1. INTRODUCTION

Proposition: Any declarative statement with a truth value assigned to it is called proposition. Ex: 1) My name is Ravindra Babu Ravula.

2) I am hungry [yes/no is truth value]

3) $x = y+1$ [No truth value]

2. CONNECTIVES ^ AND v AND ~

Propositional variable: Variable used to denote a proposition is called pv.

Connectives: These are the things which connect more than one propositional variable (propositions)

p: My name is RBR

Negation: ~p: My name is not RBR

: It is not the case that my name is not RBR.

$\wedge$: p: I got 80% marks
Q: I got A Grade
$p \wedge Q$ = I got 80% marks AND I got 'A' grade.

$\vee$: P:
Q:
$p \vee Q$ = I got 80% marks OR A grade.

P.	~p.
0	1
1	0

P	Q	P∧Q.
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	PvQ
T	T	T
F	T	T
T	F	T
F	F	F.

3. IMPLICATION

P	Q	P→Q.
T	T	T
T	F	F
F	T	T
F	F	T

p: you get 60 marks in Gate

q: you will goto iit's.

p→Q: If you get 60 marks in Gate then you will goto IIT's.

p: Today is monday (T/F)

Q: $5 \times 2 = 10$ (T)

P	Q	
T	T	→T
F	T	→T

∴ p→Q is always a

The various ways of Representing $p \to Q$ are:

1> If p then Q.
2> 'p' is sufficient for Q.
3> Q if 'p'.
4> Q when 'p'.

→ A necessary condition for 'p' is Q.
→ Q unless $\sim p$
→ 'p' implies Q.
→ 'p' only if Q.
→ A sufficient condition for 'Q' is 'p'.
→ 'Q' whenever 'p'.
→ 'Q' is necessary for 'p'.
→ 'Q' follows from 'p'.

4. QUESTION ON IMPLICATION

$p \to Q$: If today is friday then $2 \times 3 = 6$.

Converse: $Q \to p$: If $2 \times 3 = 6$ then today is friday

Contrapositive: $\sim Q \to \sim p$: If $2 \times 3 \neq 6$ then today is not friday

Inverse: $\sim p \to \sim Q$: If today is not friday then $2 \times 3 \neq 6$.

i> $p \to Q \neq \quad \to p$.

$p \to Q \cong [\sim p \vee Q]$

$\cong (\sim Q \to \sim p)$

$p \to Q \cong \sim Q \to \sim p$ (conditional & contrapositive)

$q \to p \cong \sim p \to \sim Q$ (Inverse and Converse)

5. BI- CONDITIONAL

P	Q	$P \Leftrightarrow Q$	$(P \to Q) \wedge (Q \to P)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

$\therefore p \Leftrightarrow Q \cong (p \to Q) \wedge (Q \to p)$

Various Representations of Bi conditional are: if and only iff
iff
it is necessary and sufficient

precedence of operations : 1> Negation
2> Conjunction
3> Disjunction
4> Implication
5> Bi Implication

$\Rightarrow \sim p \wedge q \vee i$

$\Rightarrow ((\sim p) \wedge q) \vee i$ //.

6. EXAMPLE 1

you can acess the internet only if you are a computer science major or you are not a freshman.

(31)

There are 3 statements here

p: you can acess the internet $\rightarrow p$.

q: you are a computer science major

R: you are not a freshman.

$\left.\begin{array}{l} q \\ r \end{array}\right\} (q \vee r)$

$$\boxed{p \rightarrow (q \vee r)}$$

$\Rightarrow$ what ever before "only if" is called Hypothesis and what is given after that is called conclusion.

7. EXAMPLE 2

you cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years.

$\sim p$ ... Q ... r

P: you can ride the roller coaster

q: you are under 4 feet tall

r: you are older than 16 years.

Now, we can replace "unless" with "And" and Negate the statement "r".

$\therefore \quad (Q \wedge \sim r)$

$p \rightarrow Q$ (r if p)

conclusion Hypothesis (After if)

$(\sim p)$ $(Q \wedge \sim r)$ $\rightarrow$ present after if $\rightarrow$ Hypothesis.

Before if = conclusion

$\therefore$ Final result = Hypothesis $\rightarrow$ Conclusion

$$\Rightarrow \boxed{(Q \wedge \sim r) \rightarrow p.}$$

8. EXAMPLE - 3

$\underbrace{\text{The automated reply cannot be sent}}_{\sim p}$ when $\underbrace{\text{the file system is full}}_{Q}$

$\sim p$ when q.

p: The automated reply can be sent

Q: The file system is full

$\therefore \boxed{q \rightarrow \sim p}$

if $p \rightarrow Q$. = Q when P

Conclusion Hypothesis

9. CONSISTENT SYSTEM

Determine whether these specifications are consistent.

→ The diagnostic message is stored in the buffer or it is retransmitted

→ The diagnostic message is not stored in the buffer

→ "If the diagnostic message is stored in the buffer, then it is retransmitted.

1> $\underbrace{\text{The diagnostic message is stored in the buffer}}_{p}$ or $\underbrace{\text{it is retransmitted}}_{Q}$ $= p \vee Q$.

2> The diagnostic message is not stored in buffer $(\neg p)$

3> If the diagnostic message is stored in the buffer, then it is retransmitted. $(p \rightarrow Q)$

$\therefore$ 1> $(p \vee Q)$

2> $(\sim p)$

3> $(p \rightarrow Q)$

Now we should find some assignment of truth values to 'p' and 'Q' such that all the '3' will be True simultaneously

$p \vee Q$. $\quad \sim p \quad$ $p \rightarrow Q$.

This should be true $\Rightarrow \sim p = T$

$\Rightarrow \boxed{p = F}$

Now, $p \vee Q$ should be True $\Rightarrow \boxed{P = F}$ already known

$\boxed{q = T}$

10. EQUIVALENCES

1> $p \to Q = \sim p \vee Q$

2> $p \to Q = \sim q \to \sim p$

3> $p \vee Q = \sim p \to Q$

4> $p \wedge Q = \sim (Q \to \sim p)$

5) $\sim (p \to Q) = p \wedge \sim q$

6) $(p \to Q) \wedge (p \to r) = p \to (Q \wedge r)$

7> $(p \to r) \wedge (q \to r) = (p \vee Q) \to r$

8> $(p \to Q) \vee (p \to r) = p \to (q \vee r)$

9> $(p \to r) \vee (q \to r) = (p \wedge Q) \to r$

10> $(p \leftrightarrow q) = (p \to q) \wedge (q \to p)$

11> $\sim (p \leftrightarrow q) \equiv (p \leftrightarrow \sim q)$

11. DE MORGANS LAW

1> $\sim (p \vee Q) \equiv \sim p \wedge \sim Q$.

2> $\sim (p \wedge Q) = \sim p \vee \sim Q$.

Ex: Ravi has a computer and a phone

⇓

If i apply Negation to this what will be the output??

↓

let p: Ravi has a computer

Q: Ravi has a phone

Now, $\sim (p \wedge Q) = \sim p \vee \sim Q$.

→ Ravi doesnot have a ~~phone~~ computer or Ravi doesnot have a phone

12. PROPOSITIONAL FUNCTION

→ proposition is a statement to which we can assign a truth value.

Ex: 2 is greater than 10 (False) ← proposition.

$\{\underbrace{x}_{\text{variable}} \underbrace{\text{ is greater than 10}}_{\text{predicate}}\}$ ← cannot take T/F unless the value of x is known.

↓

This is called propositional Function. $p(x)$

∴ $p(x)$: x is greater than 10

$p(2)$: 2 is >10 (F)

$p(12)$: 12 is >10 (T)

→ The propositional function transforms itself into proposition.

$p(x)$ $p(x,y)$ computer x is connected to network y.

13 QUANTIFIERS

$p(x): x > 2$ (34)

Domain: $\{1,2,3,4\}$ $p(1)=F$, $p(2)=F$ $p(3)=T$, $p(4)=T$

Now, $\forall x\, p(x)$ is true, if $p(x)$ is true for all values of x.

Universal Quantifier / universal Quantification.

$\therefore$ $\boxed{\forall x p(x) = \text{False}}$ in this case

Now, let $p(x) = x > 2$

Domain $= \{1,2,3,4\}$

$p(1)=F$ $p(2)=F$ $p(3)=T$, $p(4)=T$ $\therefore$ $\exists x\, p(x)$ is true if there exists atleast one value of x in the domain for which $p(x)=$ True

In this case $\boxed{\exists x p(x) = \text{True}}$

Existential Quantifier.

14. RELATION BETWEEN THE TWO QUANTIFIERS

If $\forall x p(x)$ is true then $\exists x\, p(x)$ is True

If $\forall x p(x)$ is false then $\exists x\, p(x)$ is (false/True)

If $\exists x p(x)$ is true then $\forall x p(x)$ is true

If $\exists x p(x)$ is false then $\forall x p(x)$ is false

$p(x) = x > 3$

Domain $= \{2,3,4\}$

$\exists x p(x) = T$

$\forall x p(x) = F$

→ The precedence of $\forall, \exists$ will be higher than any other logical connectives.

$$\forall x p(x) \vee Q(x) \Rightarrow [\forall x p(x)] \vee Q(x).$$

15. DISTRIBUTING QUANTIFIERS

1> $\forall x(x+y+z=10) \cong (\forall x(x)) + y + z = 10$

2> $\exists x p(x) \vee \forall x Q(x) \cong [\exists x p(x)] \vee [\forall x Q(x)]$

3> $\forall x\,(p(x) \wedge Q(x))$

Domain $= \{x_1 x_2\}$ $= p(x_1) \wedge Q(x_1) \wedge Q(x_2) \wedge Q(x_2)$ $\therefore \forall$ can be

④ $\forall x$ cannot be "distributed over disjunction." ⑤ 35

$\forall x\,(p(x) \vee Q(x)) = [p(x_1) \vee Q(x_1)] \wedge [p(x_2) \vee Q(x_2)]$

D: $\{x_1, x_2\}$ — Not Equival

$[\forall x\, p(x)] \vee [\forall x\, Q(x)] = [p(x_1) \wedge p(x_2)] \vee [Q(x_1) \wedge Q(x_2)]$

⑤ $\exists x(p(x) \vee Q(x)) = [\exists x\, p(x)] \vee [\exists x\, Q(x)]$ → There exists can be distributive over disjunction.

16. QUANTIFIERS WITH NEGATION

1) Every student in the class has taken a course in CS

$p(x)$: x has taken a course in CS

$\forall x\, p(x)$:

x = students of the class.

Now, $\sim$(statement 1) = There exist atleast one student in the class who has not taken the course in CS.

$$\boxed{\sim(\forall x\, p(x)) \equiv \exists x(\sim p(x))} \qquad \boxed{\sim \exists x\, p(x) \equiv \forall x \sim p(x)}$$

17. EXAMPLES ON NEGATING THE QUANTIFIERS

→ There is an honest politician

$H(x)$: x is honest

$\exists x\, H(x)$: There is an honest politician

x: {All politicians}

Now, $\sim(\exists x\, H(x)) = \forall x \sim (H(x))$

= All the politicians are dishonest.

→ All Americans eat cheese burgers.

$C(x)$: x eats cheese burgers ⇒ $\forall x\, C(x)$ and x: {All Americans}, Now

$\sim(\forall x\, C(x)) = \exists x \sim (C(x))$ ⇒ "There exists an American who dont eat cheese burgers."

18. IMPORTANT EXAMPLES

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1> $\sim \forall x(p(x) \wedge Q(x))$

$\Rightarrow \sim \forall(x)\,[R(x)]$ where $R(x) = p(x) \wedge Q(x)$

$\Rightarrow \exists x \; \sim(R(x))$

$\Rightarrow \exists x \; \sim(p(x) \wedge Q(x))$

$\Rightarrow \exists x\,[\sim p(x) \vee \sim Q(x)]$

2> $\sim \forall x(p(x) \to Q(x))$

$\Rightarrow \sim \forall x(\sim p(x) \vee Q(x))$

$\Rightarrow \exists x\,[p(x) \wedge \sim Q(x)]$

Statement 4

OPA : 3> $\sim \exists x\,(p(x) \wedge Q(x)) = \forall x(\sim p(x) \vee \sim Q(x)) \equiv \forall x\,(p(x) \to \sim Q(x))$

4> $\forall x\,(p(x) \to \sim Q(x)) = \forall x\,(\sim p(x) \vee \sim Q(x))$

19. TRANSLATING ENGLISH STATEMENTS TO PROPOSITIONAL FUNCTIONS

Every student in the class has taken the course operating system.

$O(x)$: x has taken the course OS.

x : {students in the class}

$\forall x\, O(x)$

Now let $S(x)$: x is a student in the class.

$\therefore$ If 'x' is a student in the class then he has taken OS

$\forall x(S(x) \to O(x))$. (domain is expanded here)

For all people in the world — if he has a student in the class then he has taken the course OS.

x = {All the people in the world}

Hint: Now, if you see the word Every in the Question the option Ans must contain $\forall$ and $\to$.

Now, $\sim \forall x(S(x) \to O(x)) = \exists x(S(x) \wedge \sim O(x))$ → Check

20. TRANSLATION CONTINUED

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Some student in the class has taken OS course

$O(x)$: x has taken CS.

$\Rightarrow \exists x \, O(x)$

x: students in the class

Now, $S(x)$: x is student in the class

$\exists x \, (S(x) \wedge O(x))$ $\Rightarrow$ Now, if you see "some" then Ans should contain ($\exists$ and AND).

Now, $\sim [\exists x (S(x) \wedge O(x)]$

$\sim$(some student in the class has taken OS course) = (No student in the class has taken OS course)

$\therefore \sim [\exists x \, (S(x) \wedge O(x)]$

$= \forall x \, [\sim S(x) \vee \sim O(x)]$

$= \forall x \, [S(x) \rightarrow \sim O(x)]$